

Calculus 1-2

1. a) $x^3 - 1$
 $(x-1)(x^2+x+1)$

b) $t^3 + 64$
 $t^3 + 4^3$
 $(t+4)(t^2-4t+4^2)$
 $(t+4)(t^2-4t+16)$

c) $8c^3 - 27d^3$
 $(2c)^3 - (3d)^3$
 $(2c-3d)[(2c)^2 + (2c)(3d) + (3d)^2]$
 $(2c-3d)(4c^2 + 6cd + 9d^2)$

d) $x^6 + 8$
 $(x^2)^3 + 2^3$
 $(x^2+2)[(x^2)^2 - x^2 \cdot 2 + 2^2]$
 $(x^2+2)(x^4 - 2x^2 + 4)$

e) $x^3 - x^2 - 16x + 16$
 $x^2(x-1) - 16(x-1)$
 $(x-1)(x^2 - 16)$
 $(x-1)(x-4)(x+4)$

f) $x^3 - 7x + 6$ factors of 6: 1, 2, 3, 6, -1, -2, -3, -6
 try 2: $2^3 - 7(2) + 6$
 $8 - 14 + 6$
 0

$x-2$ is a factor

$$\begin{array}{r} x^2 + 2x - 3 \\ x-2 \overline{) x^3 + 0x^2 - 7x + 6} \\ \underline{x^3 - 2x^2} \\ 2x^2 - 7x \\ \underline{2x^2 - 4x} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

So: $x^3 - 7x + 6 =$
 $(x-2)(x^2 + 2x - 3)$
 $(x-2)(x+3)(x-1)$

1-2 cont.

g) $x^3 + 5x^2 - 2x - 24$

try -2:

$$(-2)^3 + 5(-2)^2 - 2(-2) - 24$$

$$-8 + 20 + 4 - 24$$

-4 Not zero

try 2:

$$2^3 + 5(2)^2 - 2(2) - 24$$

$$8 + 20 - 4 - 24$$

0

$x-2$ is a factor

$$x^2 + 7x + 12$$

$$x-2 \overline{) x^3 + 5x^2 - 2x - 24}$$

$$\underline{x^3 - 2x^2}$$

$$7x^2 - 2x$$

$$\underline{7x^2 - 14x}$$

$$12x - 24$$

$$\underline{12x - 24}$$

0

So: $x^3 + 5x^2 - 2x - 24 =$

$$(x-2)(x^2 + 7x + 12)$$

$$(x-2)(x+3)(x+4)$$

h) $x^3 + 2x^2 - 11x - 12$

factors of -12:

try 3:

$$3^3 + 2(3)^2 - 11(3) - 12$$

$$27 + 18 - 33 - 12$$

0

$x-3$ is a factor

$$x^2 + 5x + 4$$

$$x-3 \overline{) x^3 + 2x^2 - 11x - 12}$$

$$\underline{x^3 - 3x^2}$$

$$5x^2 - 11x$$

$$\underline{5x^2 - 15x}$$

$$4x - 12$$

$$\underline{4x - 12}$$

0

1-2 cont.

h) cont.

$$\text{So: } x^3 + 2x^2 - 11x - 12 =$$

$$(x-3)(x^2 + 5x + 4) =$$

$$(x-3)(x+4)(x+1)$$

i) $4x^3 + 12x^2 + 5x - 6$

factors of -6:

try -3:

1, 2, 3, 6, -1, -2, -3, -6

$$4(-3)^3 + 12(-3)^2 + 5(-3) - 6$$

$$4(-27) + 12(9) - 15 - 6$$

$$-108 + 96 - 15 - 6$$

$$-33 \text{ not zero}$$

try -2:

$$4(-2)^3 + 12(-2)^2 + 5(-2) - 6$$

$$4(-8) + 12(4) - 10 - 6$$

$$-32 + 48 - 10 - 6$$

0

factor is $x - (-2) = x + 2$

$$x+2 \overline{) 4x^3 + 12x^2 + 5x - 6}$$

$$4x^3 + 8x^2$$

$$4x^2 + 5x$$

$$4x^2 + 8x$$

$$-3x - 6$$

$$-3x - 6$$

0

$$4x^2 + 4x - 3$$

$$m = -12$$

$$4x^2 + 6x - 2x - 3$$

$$a = 4$$

$$2x(2x+3) - 1(2x+3)$$

$$b = -2$$

$$(2x+3)(2x-1)$$

$$\text{So: } 4x^3 + 12x^2 + 5x - 6 =$$

$$(x+2)(4x^2 + 4x - 3)$$

$$(x+2)(2x+3)(2x-1)$$

1-2 cont.

j) $x^4 - 3x^3 - 7x^2 + 27x - 18$

factors of -18:

try 3:

1, 2, 3, 6, 9, 18

$3^4 - 3(3)^3 - 7(3)^2 + 27(3) - 18$

-1, -2, -3, -6, -9, -18

$81 - 81 - 63 + 81 - 18$

0

$x - 3$ is a factor

$$\begin{array}{r}
 x^3 \\
 x-3 \overline{) x^4 - 3x^3 - 7x^2 + 27x - 18} \\
 \underline{x^4 - 3x^3} \\
 -7x^2 + 27x \\
 \underline{-7x^2 + 21x} \\
 6x - 18 \\
 \underline{6x - 18} \\
 0
 \end{array}$$

$-7x^2 + 27x$

$-7x^2 + 21x$

$6x - 18$

$6x - 18$

0

$x^3 - 7x + 6$

So start process over

$x^3 - 7x + 6$

factors of 6: 1, 2, 3, 6, -1, -2, -3, -6

try -3: $(-3)^3 - 7(-3) + 6$

$-27 + 21 + 6$

0

so $x - (-3) = x + 3$ is a factor

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 x+3 \overline{) x^3 + 0x^2 - 7x + 6} \\
 \underline{x^3 + 3x^2} \\
 -3x^2 - 7x \\
 \underline{-3x^2 - 9x} \\
 2x + 6 \\
 \underline{2x + 6} \\
 0
 \end{array}$$

$x^3 + 3x^2$

$-3x^2 - 7x$

$-3x^2 - 9x$

$2x + 6$

$2x + 6$

0

So: $x^4 - 3x^3 - 7x^2 + 27x - 18 =$

$(x-3)(x^3 - 7x + 6) =$

$(x-3)(x+3)(x^2 - 3x + 2)$

$(x-3)(x+3)(x-2)(x-1)$

1-2 cont.

$$\begin{aligned} \text{k)} & X^{\frac{3}{2}} - X^{\frac{1}{2}} \\ & X^{\frac{1}{2}}(X^2 - 1) \\ & X^{\frac{1}{2}}(X-1)(X+1) \end{aligned}$$

$$\begin{aligned} \text{l)} & X+5 + 6X^{-1} \\ & X^{-1}(X^2 + 5X + 6) \\ & X^{-1}(X+2)(X+3) \end{aligned}$$

$$\begin{aligned} \text{m)} & X^{\frac{3}{2}} + 2X^{\frac{1}{2}} - 8X^{-\frac{1}{2}} \\ & X^{-\frac{1}{2}}(X^2 + 2X - 8) \\ & X^{-\frac{1}{2}}(X+4)(X-2) \end{aligned}$$

$$\begin{aligned} \text{n)} & 2X^{\frac{3}{2}} - 2X^{\frac{1}{2}} \\ & 2X^{\frac{1}{2}}(X^3 - 1) \\ & 2X^{\frac{1}{2}}(X-1)(X^2 + X + 1) \end{aligned}$$

$$\begin{aligned} \text{o)} & 1 + 2X^{-1} + X^{-2} \\ & X^{-2}(X^2 + 2X + 1) \\ & X^{-2}(X+1)(X+1) \end{aligned}$$

$$\begin{aligned} \text{p)} & (X^2+1)^{\frac{1}{2}} + 3(X^2+1)^{-\frac{1}{2}} \\ & (X^2+1)^{-\frac{1}{2}} [(X^2+1) + 3] \\ & (X^2+1)^{-\frac{1}{2}} (X^2+4) \end{aligned}$$