

10-3 Addition and Subtraction Formulas

1. a) $\cos 2a \cos a - \sin 2a \sin a =$
 $\cos(2a+a) =$
 $\cos 3a$

b) $\cos x \cos y + \sin x \sin y =$
 $\cos(x-y) =$
 $\cos(-3x) =$
 $\cos(3x)$

c) $\sin 5 \cos 2 - \cos 5 \sin 2 =$
 $\sin(5-2) =$
 $\sin 3$

d) $\frac{\tan 2a + \tan 3a}{1 - \tan 2a \tan 3a} =$
 $\tan(2a+3a) =$
 $\tan 5a$

2. a) $\sin \frac{11\pi}{12} =$
 $\sin \left(\frac{2\pi}{12} + \frac{3\pi}{12} \right) =$
 $\sin \left(\frac{2\pi}{3} + \frac{\pi}{4} \right) =$
 $\sin \frac{2\pi}{3} \cos \frac{\pi}{4} + \cos \frac{2\pi}{3} \sin \frac{\pi}{4} =$
 $\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{-1}{2} \cdot \frac{\sqrt{2}}{2} =$
 $\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} =$
 $\frac{\sqrt{6} - \sqrt{2}}{4}$

b) $\cos \frac{13\pi}{12} =$
 $\cos \frac{4\pi}{12} \cos \frac{9\pi}{12} - \sin \frac{4\pi}{12} \sin \frac{9\pi}{12} =$
 $\cos \frac{\pi}{3} \cos \frac{3\pi}{4} - \sin \frac{\pi}{3} \sin \frac{3\pi}{4} =$
 $\frac{1}{2} \cdot \left(-\frac{\sqrt{2}}{2} \right) - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} =$
 $-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} =$
 $-\frac{\sqrt{2} + \sqrt{6}}{4}$

10-3 cont.

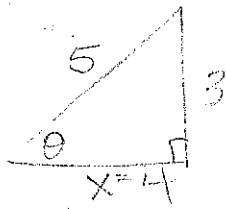
$$\begin{aligned} 2. c) \tan\left(-\frac{7}{12}\pi\right) &= \\ \tan\left(-\frac{4\pi}{12} - \frac{3\pi}{12}\right) &= \\ \tan\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) &= \\ \frac{\tan\left(-\frac{\pi}{3}\right) + \tan\left(-\frac{\pi}{4}\right)}{1 - \tan\left(-\frac{\pi}{3}\right)\tan\left(-\frac{\pi}{4}\right)} &= \\ \frac{-\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 - (-\tan\frac{\pi}{3})(-\tan\frac{\pi}{4})} &= \\ \frac{-\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}} &= \\ \frac{-\sqrt{3} - 1}{1 - \sqrt{3} \cdot 1} &= \\ \frac{-\sqrt{3} - 1}{1 - \sqrt{3}} & \end{aligned}$$

$$\begin{aligned} 3. a) \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right) &= \\ \sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3} &= \\ \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} &= \\ \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} &= \\ \frac{\sqrt{2} - \sqrt{6}}{4} & \end{aligned}$$

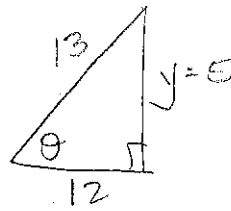
$$\begin{aligned} c) \tan\left(-\frac{3\pi}{4} + \frac{2\pi}{3}\right) &= \\ \tan\left(\frac{2\pi}{3} - \frac{3\pi}{4}\right) &= \\ \frac{\tan\frac{2\pi}{3} - \tan\frac{3\pi}{4}}{1 + \tan\frac{2\pi}{3}\tan\frac{3\pi}{4}} &= \\ \frac{\sqrt{3} - (-1)}{1 + (-\sqrt{3})(-1)} &= \\ \frac{-\sqrt{3} + 1}{1 + \sqrt{3}} &= \\ \frac{1 - \sqrt{3}}{1 + \sqrt{3}} & \end{aligned}$$

$$\begin{aligned} b) \cos\left(-\frac{\pi}{6} - \frac{\pi}{4}\right) &= \\ \cos\left[-\left(\frac{\pi}{6} + \frac{\pi}{4}\right)\right] &= \\ \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) &= \\ \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4} &= \\ \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} &= \\ \frac{\sqrt{6} - \sqrt{2}}{4} &= \\ \frac{\sqrt{6} - \sqrt{2}}{4} & \end{aligned}$$

10-3 cont
 4. interval $(0, \frac{\pi}{2})$, $\sin x = \frac{3}{5}$, $\cos y = \frac{12}{13}$
 Q1 \rightarrow



$$\begin{aligned} 3^2 + x^2 &= 5^2 \\ 9 + x^2 &= 25 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

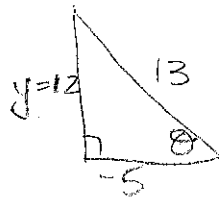


$$\begin{aligned} 12^2 + y^2 &= 13^2 \\ 144 + y^2 &= 169 \\ y^2 &= 25 \\ y &= 5 \end{aligned}$$

$$\begin{aligned} \text{a) } \sin(x-y) &= \sin x \cos y - \cos x \sin y \\ &= \frac{3}{5} \cdot \frac{12}{13} - \frac{4}{5} \cdot \frac{5}{13} \\ &= \frac{36}{65} - \frac{20}{65} \\ &= \frac{16}{65} \end{aligned}$$

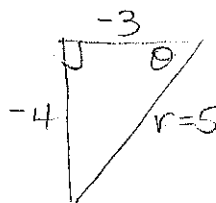
$$\begin{aligned} \text{b) } \cos(x+y) &= \cos x \cos y - \sin x \sin y \\ &= \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13} \\ &= \frac{48}{65} - \frac{15}{65} \\ &= \frac{33}{65} \end{aligned}$$

5. interval $(\frac{\pi}{2}, \pi)$ Q2
 $\cos x = -\frac{5}{13}$



$$\begin{aligned} 5^2 + y^2 &= 13^2 \\ 25 + y^2 &= 169 \\ y^2 &= 144 \\ y &= 12 \end{aligned}$$

interval $(\pi, \frac{3\pi}{2})$ Q3
 $\tan y = \frac{4}{3}$



$$\begin{aligned} 3^2 + 4^2 &= r^2 \\ 9 + 16 &= r^2 \\ 25 &= r^2 \\ 5 &= r \end{aligned}$$

$$\begin{aligned} \text{a) } \sin(x+y) &= \sin x \cos y + \cos x \sin y \\ &= \frac{12}{13} \cdot \frac{-3}{5} + \frac{-5}{13} \cdot \frac{-4}{5} \\ &= \frac{-36}{65} + \frac{20}{65} \\ &= \frac{-16}{65} \end{aligned}$$

10-3 cont.

$$\begin{aligned} 5. b) \tan(x-y) &= \frac{\tan x - \tan y}{1 + \tan x \tan y} \\ &= \frac{\frac{12}{-5} - \frac{4}{3}}{1 + \frac{12}{-5} \left(\frac{4}{3}\right)} \\ &= \frac{\frac{-36}{15} - \frac{20}{15}}{\frac{15}{15} + \frac{-48}{15}} \\ &= \frac{-56}{-33} \\ &= \frac{56}{33} \end{aligned}$$

$$\begin{aligned} 6. a) \sin 50^\circ \cos 20^\circ - \cos 50^\circ \sin 20^\circ &= \sin(50^\circ - 20^\circ) \\ &= \sin 30^\circ \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} b) \sin \frac{5\pi}{36} \cos \frac{5\pi}{18} + \cos \frac{5\pi}{36} \sin \frac{5\pi}{18} &= \sin\left(\frac{5\pi}{36} + \frac{5\pi}{18}\right) \\ &= \sin\left(\frac{5\pi}{36} + \frac{10\pi}{36}\right) \\ &= \sin \frac{15\pi}{36} \\ &= \sin \frac{5\pi}{12} \end{aligned}$$

$$\begin{aligned} \sin \frac{5\pi}{12} &= \sin\left(\frac{8\pi}{12} - \frac{3\pi}{12}\right) \\ &= \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) \\ &= \sin \frac{2\pi}{3} \cos \frac{\pi}{4} - \cos \frac{2\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{1}{2}\right) \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} 7. \sin(a-b) &= \sin[a+(-b)] \\ &= \sin a \cos(-b) + \cos a \sin(-b) \\ &= \sin a \cos b + \cos a (-\sin b) \\ &= \sin a \cos b - \cos a \sin b \end{aligned}$$

$$\begin{aligned} \sin(-b) &= -\sin b \\ \cos(-b) &= \cos b \end{aligned}$$

10-3 cont

$$\begin{aligned}
 8. \tan(a-b) &= \frac{\sin(a-b)}{\cos(a-b)} \\
 &= \frac{\sin a \cos b - \cos a \sin b}{\cos a \cos b + \sin a \sin b} \\
 &= \frac{\sin a \cos b - \cos a \sin b}{\cos a \cos b} \\
 &= \frac{\cos a \cos b + \sin a \sin b}{\cos a \cos b} \\
 &= \frac{\frac{\sin a \cos b}{\cos a \cos b} - \frac{\cos a \sin b}{\cos a \cos b}}{\frac{\cos a \cos b}{\cos a \cos b} + \frac{\sin a \sin b}{\cos a \cos b}} \\
 &= \frac{\tan a - \tan b}{1 + \tan a \tan b}
 \end{aligned}$$

$$\begin{array}{l|l}
 9. a) & \cos\left(\frac{3\pi}{2} + x\right) = \sin x \\
 \cos \frac{3\pi}{2} \cos x - \sin \frac{3\pi}{2} \sin x & \\
 0 \cdot \cos x - (-1) \sin x & \\
 0 + \sin x & \\
 \sin x &
 \end{array}$$

$$\begin{array}{l|l}
 b) & \tan(2\pi - x) = -\tan x \\
 \frac{\tan 2\pi - \tan x}{1 + \tan 2\pi \tan x} & \\
 \frac{0 - \tan x}{1 + 0 \cdot \tan x} & \\
 \frac{-\tan x}{1} & \\
 -\tan x &
 \end{array}$$

10-3 cont.

9. c)

$$\tan\left(\frac{\pi}{2} + x\right)$$

$$\frac{\sin\left(\frac{\pi}{2} + x\right)}{\cos\left(\frac{\pi}{2} + x\right)}$$

$$\frac{\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x}{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x}$$

$$\frac{1 \cdot \cos x + 0 \cdot \sin x}{0 \cdot \cos x - 1 \cdot \sin x}$$

$$\frac{\cos x}{-\sin x}$$

$$= -\cot x$$

$$= -\cot x$$

d) $-\tan(-x - \pi)$
 $= -\tan[-(x + \pi)]$
 $= -[-\tan(x + \pi)]$

$$\frac{\tan(x + \pi)}{\tan x + \tan \pi}$$

$$\frac{\tan x + 0}{1 - \tan x \cdot 0}$$

$$\frac{\tan x}{1}$$

$$= \tan x$$

$$= \tan x$$