

10-4 Double Angle Formulas

$$1. a) \cos 2(2x) = \cos^2 2x - \sin^2 2x$$

$$\text{OR} = 2\cos^2 2x - 1$$

$$\text{OR} = 1 - 2\sin^2 2x$$

$$b) \sin 3x = \sin 2(1.5x)$$

$$= 2\sin 1.5x \cos 1.5x$$

$$c) \tan 6x = \tan 2(3x)$$

$$= \frac{2 \tan 3x}{1 - \tan^2 3x}$$

$$2. a) 2 \sin 3\theta \cos 3\theta = \sin 2(3\theta)$$

$$= \sin 6\theta$$

$$b) 6 \sin \theta \cos \theta = 3(2 \sin \theta \cos \theta)$$

$$= 3(\sin 2\theta)$$

$$= 3 \sin 2\theta$$

$$c) \cos^2 \frac{3\theta}{2} - \sin^2 \frac{3\theta}{2} = \cos 2\left(\frac{3\theta}{2}\right)$$

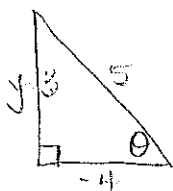
$$= \cos 3\theta$$

$$d) 1 - 2 \sin^2 \frac{\theta}{4} = \cos 2\left(\frac{\theta}{4}\right)$$

$$= \cos \frac{2\theta}{4}$$

$$= \cos \frac{\theta}{2}$$

$$3. \cos \theta = -\frac{4}{5}, \quad \frac{\pi}{2} \leq \theta \leq \pi \text{ (Q2)}$$



$$4^2 + y^2 = 5^2$$

$$16 + y^2 = 25$$

$$y^2 = 9$$

$$y = 3$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{3}{5}\right) \left(-\frac{4}{5}\right)$$

$$= -\frac{24}{25}$$

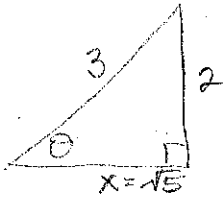
\cos is pos, \sin is neg

$\frac{\sin}{\cos}$	All+
$\frac{\cos}{\sin}$	sin-
$\frac{\cos}{\sin}$	cos+

so it has to be Q4.

10-4 cont.

4. $\sin \theta = \frac{2}{3}$, $0 \leq \theta \leq \frac{\pi}{2}$ (Q1)

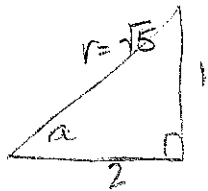


$$\begin{aligned} 2^2 + x^2 &= 3^2 \\ 4 + x^2 &= 9 \\ x^2 &= 5 \\ x &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} \sin 4\theta &= \sin 2(2\theta) \\ &= 2 \sin 2\theta \cos 2\theta \\ &= \frac{2}{1} \left(\frac{4\sqrt{5}}{9} \right) \left(\frac{1}{9} \right) \\ &= \frac{8\sqrt{5}}{81} \end{aligned}$$

$\sin 2\theta = 2 \sin \theta \cos \theta$
 $= \frac{2}{1} \cdot \frac{2}{3} \left(\frac{\sqrt{5}}{3} \right)$
 $= \frac{4\sqrt{5}}{9}$
 $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
 $= \left(\frac{\sqrt{5}}{3} \right)^2 - \left(\frac{2}{3} \right)^2$
 $= \frac{5}{9} - \frac{4}{9}$
 $= \frac{1}{9}$

5. $\tan a = \frac{1}{2}$, $0 \leq a \leq \frac{\pi}{2}$ (Q1)



$$\begin{aligned} 1^2 + 2^2 &= r^2 \\ 1 + 4 &= r^2 \\ 5 &= r^2 \\ \sqrt{5} &= r \end{aligned}$$

$$\begin{aligned} \tan 2a &= \frac{2 \tan a}{1 - \tan^2 a} \\ &= \frac{2 \cdot \frac{1}{2}}{1 - \left(\frac{1}{2} \right)^2} \\ &= \frac{1}{\frac{4}{4} - \frac{1}{4}} \\ &= \frac{1}{\frac{3}{4}} \\ &= \frac{4}{3} \end{aligned}$$