

10-5 Trigonometric Identities

$$\begin{aligned}
 1. \ a) \quad \sin x \tan x &= \sec x - \cos x \\
 \frac{\sin x \sin x}{\cos x} &= \frac{1}{\cos x} - \cos x \\
 \frac{\sin^2 x}{\cos x} &= \frac{1 - \cos^2 x}{\cos x} \\
 &= \frac{\sin^2 x}{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \cos^4 x - \sin^4 x &= 1 - 2\sin^2 x \\
 (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) &= \frac{1 - 2\sin^2 x}{\cos^2 x} \\
 (\cos^2 x)(1) &= \\
 \cos^2 x &=
 \end{aligned}$$

$$\begin{aligned}
 c) \quad \frac{\tan x + \tan y}{\cot x + \cot y} &= (\tan x)(\tan y) \\
 \frac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}} &= \\
 \frac{\tan x + \tan y}{\frac{\tan y + \tan x}{\tan x \tan y}} &= \\
 \frac{\tan x + \tan y}{\frac{\tan x + \tan y}{\tan x \tan y}} &= \\
 \frac{\tan x + \tan y}{\tan x + \tan y} \cdot \frac{\tan x \tan y}{\tan x \tan y} &= \\
 \tan x \tan y &=
 \end{aligned}$$

10-5 cont.

$$\begin{aligned}
 2. a) \quad & \cos(x+y)\cos y + \sin(x+y)\sin y = \cos x \\
 & (\cos x \cos y - \sin x \sin y)\cos y + (\sin x \cos y + \cos x \sin y)\sin y \\
 & \cos x \cos^2 y - \sin x \sin y \cos y + \sin x \sin y \cos y + \cos x \sin^2 y \\
 & \cos x \cos^2 y + \cos x \sin^2 y \\
 & \cos x (\cos^2 y + \sin^2 y) \\
 & \cos x (1) \\
 & \cos x
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \cos\left(\frac{3\pi}{4} + x\right) + \sin\left(\frac{3\pi}{4} - y\right) \\
 & \cos\frac{3\pi}{4}\cos x - \sin\frac{3\pi}{4}\sin x + \sin\frac{3\pi}{4}\cos y - \cos\frac{3\pi}{4}\sin y \\
 & \left(-\frac{\sqrt{2}}{2}\right)\cos x - \left(\frac{\sqrt{2}}{2}\right)\sin x + \left(\frac{\sqrt{2}}{2}\right)\cos y - \left(-\frac{\sqrt{2}}{2}\right)\sin y \\
 & -\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos y + \frac{\sqrt{2}}{2}\sin y \\
 & 0
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \sin(x+y)\sin(x-y) \neq \cos^2 y - \cos^2 x \\
 & (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\
 & \sin^2 x \cos^2 y - \sin x \cos x \sin y \cos y + \sin x \cos x \sin y \cos y - \cos^2 x \sin^2 y \\
 & \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\
 & (1 - \cos^2 x)\cos^2 y - \cos^2 x(1 - \cos^2 y) \\
 & \cos^2 y - \cos^2 x \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y \\
 & \cos^2 y - \cos^2 x
 \end{aligned}$$

$$\begin{aligned}
 3. a) \quad & \sin\left(\frac{\pi}{2} - x\right) \cot\left(\frac{\pi}{2} + x\right) \neq -\sin x \\
 & \sin\left(\frac{\pi}{2} - x\right) \frac{\cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)} \\
 & \cos x \cdot \frac{-\sin x}{\cos x} \\
 & -\sin x
 \end{aligned}$$

10-5 cont.

$$3. b) \frac{\cos(-x) + \cos(\pi-x)}{\cos x + (-\cos x)} \\ \frac{\cos x - \cos x}{0}$$

$$\frac{\cos(\pi+x) + \cos x}{-\cos x + \cos x} \\ 0$$

$$4. a) \frac{\sin 2x}{1 + \cos 2x} \\ \frac{2 \sin x \cos x}{1 + 2 \cos^2 x - 1} \\ \frac{2 \sin x \cos x}{2 \cos^2 x} \\ \frac{\sin x}{\cos x}$$

$$\tan x \\ \frac{\sin x}{\cos x}$$

$$b) \frac{2 \csc 2x}{2} \\ \frac{1}{\sin 2x} \\ \frac{1}{2 \sin x \cos x} \\ \frac{1}{\sin x \cos x}$$

$$\sec x \csc x \\ \frac{1}{\cos x} \cdot \frac{1}{\sin x} \\ \frac{1}{\sin x \cos x}$$

$$c) \frac{2 \cot 2x}{2 \cos 2x} \\ \frac{1}{\sin 2x} \\ \frac{1}{2(\cos^2 x - \sin^2 x)} \\ \frac{1}{2 \sin x \cos x} \\ \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$\cot x - \tan x \\ \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \\ \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x} \\ \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

10-5 cont.

$$5. a) \cos x = \frac{\sin x \tan^2 x \cot^3 x}{\sin x \cdot \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^3 x}{\sin^3 x}}$$

$$\frac{\sin^2 x \cos x}{\sin^2 x}$$

$$\cos x$$

$$b) (\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$$

$$\sin x \tan x + \sin x \cot x + \cos x \tan x + \cos x \cot x$$

$$\sin x \frac{\sin x}{\cos x} + \sin x \frac{\cos x}{\sin x} + \cos x \frac{\sin x}{\cos x} + \cos x \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x}{\cos x} + \cos x + \sin x + \frac{\cos^2 x}{\sin x}$$

$$\frac{\sin^3 x}{\sin x \cos x} + \frac{\sin x \cos^2 x}{\sin x \cos x} + \frac{\sin^2 x \cos x}{\sin x \cos x} + \frac{\cos^3 x}{\sin x \cos x}$$

$$\frac{\sin^3 x + \sin^2 x \cos x + \cos^3 x + \sin x \cos^2 x}{\sin x \cos x}$$

$$\frac{\sin^2 x (\sin x + \cos x) + \cos^2 x (\cos x + \sin x)}{\sin x \cos x}$$

$$\frac{(\sin x + \cos x)(\sin^2 x + \cos^2 x)}{\sin x \cos x}$$

$$\frac{(\sin x + \cos x)(1)}{\sin x \cos x}$$

$$\frac{\sin x + \cos x}{\sin x \cos x}$$

$$\sec x + \csc x$$

$$\frac{1}{\cos x} + \frac{1}{\sin x}$$

$$\frac{\sin x}{\cos x \sin x} + \frac{\cos x}{\cos x \sin x}$$

$$\frac{\sin x + \cos x}{\cos x \sin x}$$