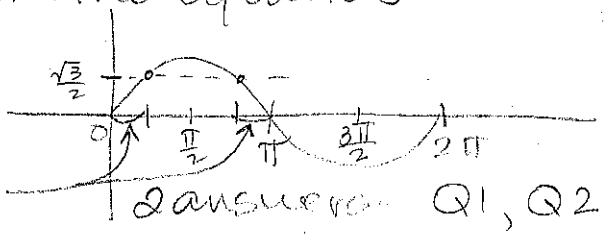
10-b Solving Trigonometric Equations

1. a) $\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}$



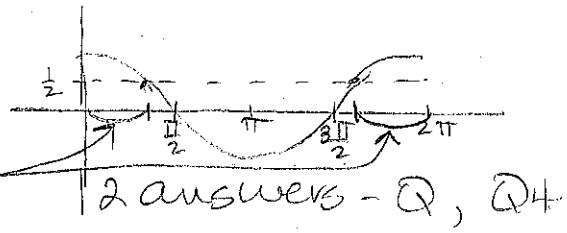
ref angle $= \frac{\pi}{3}$ — 2 answers: Q1, Q2

so $x_1 = \frac{\pi}{3}$

$x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

b) $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$



ref angle $= \frac{\pi}{3}$ — 2 answers - Q, Q4

so $x_1 = \frac{\pi}{3}$

$x_2 = 2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

2. $\sin^2 x - \sin x = 2$ $\quad [-\pi, 0]$

$\sin^2 x - \sin x - 2 = 0$

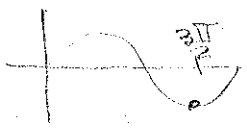
$(\sin x - 2)(\sin x + 1) = 0$

$\sin x - 2 = 0$ or $\sin x + 1 = 0$

$\sin x = 2$ $\qquad \sin x = -1$

no solution $\qquad x = \frac{3\pi}{2}$

because $\sin x > 1$



$x = \frac{3\pi}{2} - 2\pi = -\frac{\pi}{2}$ in $[-\pi, 0]$

3. $6\sin^2 x - 5\cos x - 2 = 0$ $\quad [0, 2\pi]$

$6(1 - \cos^2 x) - 5\cos x - 2 = 0$

$6 - 6\cos^2 x - 5\cos x - 2 = 0$

$-6\cos^2 x - 5\cos x + 4 = 0$

$6\cos^2 x + 5\cos x - 4 = 0$

$6\cos^2 x - 3\cos x + 8\cos x - 4 = 0$

$3\cos x(2\cos x - 1) + 4(2\cos x - 1) = 0$

$(2\cos x - 1)(3\cos x + 4) = 0$

See next page

10-6 cont.

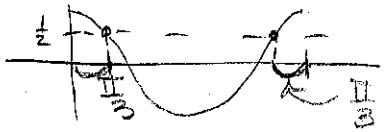
3. cont.

$2\cos x - 1 = 0$ or $3\cos x + 4 = 0$

$2\cos x = 1$ $\quad 3\cos x = -4$

$\cos x = \frac{1}{2}$ $\quad \cos x = -\frac{4}{3}$ no solution because $\cos x \geq -1$

$x = \frac{\pi}{3}$



$x_1 = \frac{\pi}{3}$

$x_2 = 2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

4. a) $\cos 2x = \cos^2 x$ $\quad [-\pi, \pi]$

$2\cos^2 x - 1 = \cos^2 x$

$\cos^2 x - 1 = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$

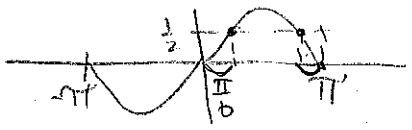
$x = -\pi, 0, \pi$

b) $\sin 2x = \cos x$ $\quad [-\pi, \pi]$

$2\sin x \cos x = \cos x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}$

$x_1 = \frac{\pi}{6}$

$x_2 = \pi - \frac{\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$