

11-2 Limits of Trigonometric Functions

$$1. a) \lim_{x \rightarrow 0} \frac{\sin 3x}{x} =$$

$$\lim_{x \rightarrow 0} \frac{3 \sin 3x}{3x} =$$

$$\lim_{x \rightarrow 0} 3 \left(\frac{\sin 3x}{3x} \right) =$$

$$3 \cdot 1 =$$

$$3$$

$$b) \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} =$$

$$\lim_{x \rightarrow 0} \frac{\frac{ax \cdot \sin ax}{ax} \cdot \frac{1}{1}}{\frac{bx \cdot \sin bx}{bx} \cdot \frac{1}{1}} =$$

$$\lim_{x \rightarrow 0} \frac{ax \left(\frac{\sin ax}{ax} \right)}{bx \left(\frac{\sin bx}{bx} \right)} =$$

$$\frac{a(1)}{b(1)} = \frac{a}{b}$$

$$c) \lim_{x \rightarrow 0} \frac{\sin^3 2x}{\sin^3 3x} =$$

$$\lim_{x \rightarrow 0} \frac{(\sin 2x)^3}{(\sin 3x)^3} =$$

$$\lim_{x \rightarrow 0} \left(\frac{\frac{2x \cdot \sin 2x}{2x} \cdot \frac{1}{1}}{\frac{3x \cdot \sin 3x}{3x} \cdot \frac{1}{1}} \right)^3 =$$

$$\lim_{x \rightarrow 0} \left[\frac{2x \left(\frac{\sin 2x}{2x} \right)}{3x \left(\frac{\sin 3x}{3x} \right)} \right]^3 =$$

$$\left[\frac{2(1)}{3(1)} \right]^3 =$$

$$\left(\frac{2}{3} \right)^3 =$$

$$\frac{8}{27}$$

$$d) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} =$$

$$\lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x)}{x(1 + \cos x)} =$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)} =$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)} =$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1 + \cos x} =$$

$$1 \cdot \frac{0}{1+1} =$$

$$0$$

11-2 cont.

1. cont.

$$e) \lim_{x \rightarrow 0} (x^2 + \cos x) =$$

$$0 + 1 =$$

1

$$f) \lim_{x \rightarrow 0} \frac{\sin 5x}{5} =$$

$$\lim_{x \rightarrow 0} \frac{x}{x} \cdot \frac{\sin 5x}{5} =$$

$$\lim_{x \rightarrow 0} x \cdot \left(\frac{\sin 5x}{5x} \right) =$$

$$0 \cdot 1 =$$

0

$$g) \lim_{x \rightarrow 0} \frac{\tan 3x}{3 \tan x} =$$

$$\lim_{x \rightarrow 0} \frac{1}{3} \frac{\tan 3x}{\tan x} =$$

$$\lim_{x \rightarrow 0} \frac{1 \cdot \frac{3x}{3x} \cdot \tan 3x}{3 \cdot \frac{0x}{0x} \cdot \tan 2x} =$$

$$\lim_{x \rightarrow 0} \frac{1 \cancel{3x} \left(\frac{\tan 3x}{3x} \right)}{3 \cancel{0x} \left(\frac{\tan 2x}{0x} \right)} =$$

$$\frac{1}{2} \cdot 1 =$$

$\frac{1}{2}$

$$h) \lim_{x \rightarrow 0} \frac{\sin^2 3x}{x^2} =$$

$$\lim_{x \rightarrow 0} \frac{(\sin 3x)^2}{x^2} =$$

$$\lim_{x \rightarrow 0} \frac{3^2}{3^2} \frac{(\sin 3x)^2}{x^2} =$$

$$\lim_{x \rightarrow 0} 9 \left(\frac{\sin 3x}{3x} \right) \left(\frac{\sin 3x}{3x} \right) =$$

$$9 \cdot 1 \cdot 1 =$$

9