11-3 Derivatives of the Sine and Cosine Functions

1. a) $y = \cos(-4x)$

$\frac{dy}{dx} = -\sin(-4x)\frac{d}{dx}(-4x)$

$= -\sin(-4x)(-4)$

$= 4\sin(-4x)$

$= 4[-\sin(4x)]$

$= -4\sin(4x)$

b) $y = \sin(3x + 2\pi)$

$\frac{dy}{dx} = \cos(3x+2\pi)(3)$

$= 3\cos(3x+2\pi)$

c) $y = 4\sin(-2x^2-3)$

$\frac{dy}{dx} = 4\cos(-2x^2-3)(-4x)$

$= -16x\cos(-2x^2-3)$

d) $y = \sin x^2$

$\frac{dy}{dx} = \cos x^2(2x)$

$= 2x\cos x^2$

e) $y = -\cos x^2$

$\frac{dy}{dx} = -(-\sin x^2)(2x)$

$= 2x\sin x^2$

f) $y = \cos(x^2-2)^2$

$\frac{dy}{dx} = -\sin(x^2-2)^2\,2(x^2-2)(2x)$

$= -4x(x^2-2)\sin(x^2-2)^2$

g) $y = x\cos x$

$\frac{dy}{dx} = x(-\sin x)(1) + (1)(\cos x)$

$= -x\sin x + \cos x$

h) $y = \frac{x}{\sin x}$

$\frac{dy}{dx} = \frac{(1)\sin x - x\cos x}{\sin^2 x}$

$= \frac{\sin x - x\cos x}{\sin^2 x}$

i) $y = \frac{\sin x}{1+\cos x}$

$\frac{dy}{dx} = \frac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$= \frac{\cos x(1+\cos x) + \sin^2 x}{(1+\cos x)^2}$

$= \frac{\cos x(1+\cos x) + (1-\cos^2 x)}{(1+\cos x)^2}$

$= \frac{\cos x(1+\cos x) + (1+\cos x)(1-\cos x)}{(1+\cos x)^2}$

$= \frac{\cancel{(1+\cos x)}(\cos x + 1 - \cos x)}{(1+\cos x)^{\cancel{2}}}$

$= \frac{1}{1+\cos x}$

j) $y = \sin(\cos x)$

$\frac{dy}{dx} = \cos(\cos x)(-\sin x)$

$= -\sin x\cos(\cos x)$

11-3 cont.

2\. a) $\sin y = \cos 2x$

$$\cos y \frac{dy}{dx} = -\sin 2x\,(2)$$

$$\cos y \frac{dy}{dx} = -2\sin 2x$$

$$\frac{dy}{dx} = \frac{-2\sin 2x}{\cos y}$$

b) $x\cos y = \sin(x+y)$

$$x(-\sin y)\frac{dy}{dx} + 1\cos y = \cos(x+y)\left(1+\frac{dy}{dx}\right)$$

$$-x\sin y \frac{dy}{dx} + \cos y = \cos(x+y) + \cos(x+y)\frac{dy}{dx}$$

$$\cos y - \cos(x+y) = \cos(x+y)\frac{dy}{dx} + x\sin y \frac{dy}{dx}$$

$$\cos y - \cos(x+y) = \frac{dy}{dx}\left[\cos(x+y) + x\sin y\right]$$

$$\frac{\cos y - \cos(x+y)}{\cos(x+y) + x\sin y} = \frac{dy}{dx}$$

c) $\sin y + y = \cos x + x$

$$\cos y \frac{dy}{dx} + 1\frac{dy}{dx} = (-\sin x)(1) + 1$$

$$\frac{dy}{dx}(\cos y + 1) = 1 - \sin x$$

$$\frac{dy}{dx} = \frac{1-\sin x}{\cos y + 1}$$

d) $\sin(\cos x) = \cos(\sin y)$

$$\cos(\cos x)(-\sin x)(1) = -\sin(\sin y)(\cos y)\frac{dy}{dx}$$

$$-\sin x\cos(\cos x) = \frac{dy}{dx}\left[-\sin(\sin y)\right](\cos y)$$

$$\frac{-\sin x\cos(\cos x)}{-\sin(\sin y)\cos y} = \frac{dy}{dx}$$

$$\frac{\sin x\cos(\cos x)}{\cos y\sin(\sin y)} = \frac{dy}{dx}$$