11-4 Derivatives of other trigonometric functions

1\. a) $y = 3\tan 2x$

$\frac{dy}{dx} = 3\sec^2(2x)(2)$

$= 6\sec^2 2x$

b) $y = \frac{1}{3}\cot 9x$

$\frac{dy}{dx} = \frac{1}{3}\left[-\csc^2(9x)\right](9)$

$= -\frac{9}{3}\csc^2(9x)$

$= -3\csc^2 9x$

c) $y = 12\sec \frac{1}{4}x$

$\frac{dy}{dx} = 12\sec(\frac{1}{4}x)\tan(\frac{1}{4}x)(\frac{1}{4})$

$= \frac{12}{4}\sec(\frac{1}{4}x)\tan(\frac{1}{4}x)$

$= 3\sec(\frac{1}{4}x)\tan(\frac{1}{4}x)$

d) $y = \frac{1}{4}\csc(-8x)$

$\frac{dy}{dx} = -\frac{1}{4}\left[-\csc(-8x)\cot(-8x)\right](-8)$

$= -\frac{8}{4}\csc(-8x)\cot(-8x)$

$= -2\csc(-8x)\cot(-8x)$

$= -2\left[-\csc(8x)\right]\left[-\cot(8x)\right]$

$= -2\csc(8x)\cot(8x)$

$\csc(-8x) = \frac{1}{\sin(-8x)} = \frac{1}{-\sin(8x)} = -\csc(8x)$

$\cot(-8x) = \frac{\sin(-8x)}{\cos(-8x)} = \frac{-\sin 8x}{\cos 8x} = -\cot(8x)$

e) $y = \tan y^2$

$\frac{dy}{dx} = \sec^2(y^2)(2x)$

$= 2x\sec^2 x^2$

f) $y = \tan^2 x$

$= (\tan x)^2$

$\frac{dy}{dx} = 2\tan x \cdot \sec^2 x$

$= 2\tan x\sec^2 x$

g) $y = \sin(\tan x)$

$\frac{dy}{dx} = \cos(\tan y)\sec^2 x$

h) $y = \tan^2(\cos x)$

$= \left[\tan(\cos x)\right]^2$

$\frac{dy}{dx} = 2\left[\tan(\cos x)\right]\left[\sec^2(\cos x)\right](-\sin x)$

$= -2\sin x\left[\tan(\cos x)\right]\left[\sec^2(\cos x)\right]$

2\. a) $\tan x + \sec y - y = 0$

$\sec^2 x + \sec y\tan y\frac{dy}{dx} - \frac{dy}{dx} = 0$

$\sec^2 x = \frac{dy}{dx} - \sec y\tan y\frac{dy}{dx}$

$\sec^2 x = \frac{dy}{dx}(1 - \sec y\tan y)$

$\frac{\sec^2 x}{1 - \sec y\tan y} = \frac{dy}{dx}$

b) $\tan 2x = \cos 3y$

$\sec^2(2x)\,2 = (-\sin 3y)\,3\frac{dy}{dx}$

$2\sec^2 2x = -3\sin 3y\frac{dy}{dx}$

$\frac{2\sec^2 2x}{-3\sin 3y} = \frac{dy}{dx}$