Calculus 2-3

1\. a) $\lim_{x\to -2} \frac{x+2}{x^2-4} = \lim_{x\to -2} \frac{\cancel{x+2}}{(\cancel{x+2})(x-2)} = \lim_{x\to -2} \frac{1}{x-2} = \frac{1}{-2-2} = \frac{1}{-4}$

b) $\lim_{x\to 1} \frac{x^2-3x+2}{x-1} = \lim_{x\to 1} \frac{(x-2)(\cancel{x-1})}{\cancel{x-1}} = \lim_{x\to 1} x-2 =$

$1-2=-1$

c) $\lim_{x\to 3} \frac{x^2-2x-3}{x^2-4x+3} = \lim_{x\to 3} \frac{(\cancel{x-3})(x+1)}{(\cancel{x-3})(x-1)} = \lim_{x\to 3} \frac{x+1}{x-1} =$

$\frac{3+1}{3-1} = \frac{4}{2} = 2$

factor on scrap paper

d) $\lim_{x\to -2} \frac{2x^2+5x+2}{x^2-2x-8} = \lim_{x\to -2} \frac{(\cancel{x+2})(2x+1)}{(x-4)(\cancel{x+2})} = \lim_{x\to -2} \frac{2x+1}{x-4} =$

$\frac{2(-2)+1}{-2-4} = \frac{-4+1}{-6} = \frac{-3}{-6} = \frac{1}{2}$

e) $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{(\cancel{x-1})(x^2+x+1)}{(\cancel{x-1})(x+1)} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} =$

$\frac{1^2+1+1}{1+1} = \frac{3}{2}$

f) $\lim_{x\to -3} \frac{x+3}{x^3+27} = \lim_{x\to -3} \frac{\cancel{x+3}}{(\cancel{x+3})(x^2-3x+9)} = \lim_{x\to -3} \frac{1}{x^2-3x+9} =$

$\frac{1}{(-3)^2-3(-3)+9} = \frac{1}{9+9+9} = \frac{1}{27}$

g) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} = \lim_{x\to 9} \frac{(\sqrt{x}+3)(\cancel{\sqrt{x}-3})}{\cancel{\sqrt{x}-3}} = \lim_{x\to 9} \sqrt{x}+3 =$

$\sqrt{9}+3 = 3+3 = 6$

x-9 can be considered a difference of squares - that's what I did!

12.3 cont.

Common denominator of $\frac{1}{x}$ and $\frac{1}{2}$ is $2x$

1. cont.

h) $\lim\limits_{x\to 2} \dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}\cdot\dfrac{2x}{2x} = \lim\limits_{x\to 2} \dfrac{\frac{2x}{x}-\frac{2x}{2}}{2x(x-2)} =$

$\lim\limits_{x\to 2} \dfrac{2-x}{2x(x-2)} = \lim\limits_{x\to 2} \dfrac{-1}{2x} = \dfrac{-1}{2\cdot 2} = -\dfrac{1}{4}$

$\dfrac{2-x}{x-2} = \dfrac{-1\cancel{(x-2)}}{\cancel{x-2}} = -1$

2. a) $\lim\limits_{h\to 0} \dfrac{(4+h)^3-64}{h} = \lim\limits_{h\to 0} \dfrac{[(\cancel{4}+h)-\cancel{4}][(4+h)^2+4(4+h)+16]}{h} =$

$\lim\limits_{h\to 0} \dfrac{\cancel{h}[(4+h)^2+4(4+h)+16]}{\cancel{h}} = \lim\limits_{h\to 0} (4+h)^2+4(4+h)+16 =$

$(4+0)^2+4(4+0)+16 = 16+16+16 = 48$

Note: the method in a, b, etc is different than that shown in the notes. You can use either method!

b) $\lim\limits_{h\to 0} \dfrac{(h-2)^2-4}{h} = \lim\limits_{h\to 0} \dfrac{[(h-2)-2][(h\cancel{-2})\cancel{+2}]}{h} =$

$\lim\limits_{h\to 0} \dfrac{(h-4)\cancel{h}}{\cancel{h}} = \lim\limits_{h\to 0} h-4 = 0-4 = -4$

c) $\lim\limits_{h\to 0} \dfrac{\frac{1}{1+h}-1}{h}\cdot\dfrac{(1+h)}{(1+h)} = \lim\limits_{h\to 0} \dfrac{1-(1+h)}{h(1+h)} = \lim\limits_{h\to 0} \dfrac{1-1-h}{h(1+h)} =$

$\lim\limits_{h\to 0} \dfrac{-\cancel{h}}{\cancel{h}(1+h)} = \lim\limits_{h\to 0} \dfrac{-1}{1+h} = \dfrac{-1}{1+0} = \dfrac{-1}{1} = -1$

d) $\lim\limits_{h\to 0} \dfrac{(2+h)^4-16}{h} = \lim\limits_{h\to 0} \dfrac{[(2+h)^2-4][(2+h)^2+4]}{h} =$

$\lim\limits_{h\to 0} \dfrac{[(\cancel{2}+h)-\cancel{2}][(2+h)+2][(2+h)^2+4]}{h} =$

see next page

2.3 cont

2. d) cont.

$$\lim_{h\to 0} \frac{\cancel{h}(h+4)\left[(2+h)^2+4\right]}{\cancel{h}} = \lim_{h\to 0} (h+4)\left[(2+h)^2+4\right] =$$

$$(0+4)\left[(2+0)^2+4\right] = 4\left[4+4\right] = 4(8) = 32$$

e) $$\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} = \lim_{h\to 0} \frac{(9+h)-9}{h(\sqrt{9+h}+3)} =$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)} = \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{\sqrt{9+0}+3} = \frac{1}{3+3} = \frac{1}{6}$$

f) $$\lim_{h\to 0} \frac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h} \cdot \frac{4(2+h)^2}{4(2+h)^2} = \lim_{h\to 0} \frac{4-(2+h)^2}{4h(2+h)^2} =$$

$$\lim_{h\to 0} \frac{4-(4+4h+h^2)}{4h(2+h)^2} = \lim_{h\to 0} \frac{\cancel{4}-\cancel{4}-4h-h^2}{4h(2+h)^2} =$$

$$\lim_{h\to 0} \frac{-4h-h^2}{4h(2+h)^2} = \lim_{h\to 0} \frac{\cancel{h}(-4-h)}{4\cancel{h}(2+h)^2} = \lim_{h\to 0} \frac{-4-h}{4(2+h)^2} =$$

$$\frac{-4-0}{4(2+0)^2} = \frac{-4}{4\cdot 4} = \frac{-4}{16} = \frac{-1}{4}$$

+8