

Calculus 2-4

1. a) $\lim_{x \rightarrow 3} \frac{1}{(x-3)^2}$ $x-3 \neq 0$ asymptote @ $x=3$ so. $x \neq 3$ limit does not exist

b) $\lim_{x \rightarrow -8} \frac{x^2+16x+64}{x+8} = \lim_{x \rightarrow -8} \frac{(x+8)(x+8)}{x+8} = \lim_{x \rightarrow -8} x+8 = -8+8 = 0$

c) $\lim_{x \rightarrow 1} \frac{x^4-1}{x-1} = \lim_{x \rightarrow 1} \frac{(x^2-1)(x^2+1)}{x-1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)(x^2+1)}{x-1} = \lim_{x \rightarrow 1} (x+1)(x^2+1) = (1+1)(1^2+1) = 2 \cdot 2 = 4$

1. cont.

d) $\lim_{x \rightarrow -1} \frac{x-1}{x^2-1} = \lim_{x \rightarrow -1} \frac{x-1}{(x-1)(x+1)} = \lim_{x \rightarrow -1} \frac{1}{x+1} = \frac{1}{-1+1} = \frac{1}{0}$ does not exist

e) $\lim_{x \rightarrow 1} \frac{x^2+x-2}{x^2-2x+1} = \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{(x-1)(x-1)} = \lim_{x \rightarrow 1} \frac{x+2}{x-1} = \frac{1+2}{1-1} = \frac{3}{0}$ does not exist

f) $\lim_{x \rightarrow -2} \frac{x^2-x-2}{x^2+3x+2} = \lim_{x \rightarrow -2} \frac{(x-2)(x+1)}{(x+2)(x+1)} = \lim_{x \rightarrow -2} \frac{x-2}{x+2} = \frac{-2-2}{-2+2} = \frac{-4}{0}$ does not exist

g) $\lim_{x \rightarrow 3} \frac{x^2-3^2}{x-3} = \lim_{x \rightarrow 3} \frac{\frac{1}{x^2} - \frac{1}{3^2} \cdot 9x^2}{x-3} = \lim_{x \rightarrow 3} \frac{9-x^2}{9x^2(x-3)} = \lim_{x \rightarrow 3} \frac{-1(3-x)(3+x)}{9x^2(x-3)} = \lim_{x \rightarrow 3} \frac{-(3+x)}{9x^2} = \frac{-(3+3)}{9 \cdot 3^2} = \frac{-6}{81} = \frac{-2}{27}$

2-4 cont.

$$\begin{aligned} \text{h) } \lim_{x \rightarrow 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x-4} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} &= \lim_{x \rightarrow 4} \frac{2-\sqrt{x}}{2\sqrt{x}(x-4)} \cdot \frac{(2+\sqrt{x})}{(2+\sqrt{x})} = \\ \lim_{x \rightarrow 4} \frac{-1}{2\sqrt{x}(x+4)(2+\sqrt{x})} &= \lim_{x \rightarrow 4} \frac{-1}{2\sqrt{x}(2+\sqrt{x})} = \\ \frac{-1}{2\sqrt{4}(2+\sqrt{4})} &= \frac{-1}{2 \cdot 2 \cdot 4} = \frac{-1}{16} \end{aligned}$$

1. cont.

$$\text{i) } \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^3 - x^2 - 4x - 4} =$$

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x-2)(x+2)} =$$

$$\lim_{x \rightarrow 1} \frac{x^2+x+1}{(x-2)(x+2)} = \frac{1^2+1+1}{(1-2)(1+2)} = \frac{3}{(-1)(3)} = \frac{3}{-3} = -1$$

$$\begin{aligned} x^3 - x^2 - 4x - 4 &= \\ x^2(x-1) - 4(x-1) &= \\ (x-1)(x^2-4) &= \\ (x-1)(x-2)(x+2) &= \end{aligned}$$

$$\text{j) } \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-x} \cdot \frac{\sqrt{x}+x}{\sqrt{x}+x} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+x)}{x-x^2} =$$

$$\lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+x)}{x(1-x)} = \lim_{x \rightarrow 1} \frac{\sqrt{x}+x}{x} = \frac{\sqrt{1}+1}{1} = \frac{1+1}{1} = 2$$

Remember: $\frac{x-1}{1-x} = -1$