

Calculus 2-6

1. a) $\lim_{x \rightarrow 0^+} \sqrt[4]{x} = \sqrt[4]{0} = 0$

b) $\lim_{x \rightarrow 3^+} \sqrt{x-3} = \sqrt{3-3} = \sqrt{0} = 0$

c) $\lim_{x \rightarrow 1^-} \sqrt{1-x} = \sqrt{1-1} = \sqrt{0} = 0$

d) $\lim_{x \rightarrow \frac{1}{2}^-} \sqrt[4]{1-2x} = \sqrt[4]{1-2(\frac{1}{2})} = \sqrt[4]{1-1} = \sqrt[4]{0} = 0$

e) $\lim_{x \rightarrow 6^+} |x-6| = |6-6| = |0| = 0$

f) $\lim_{x \rightarrow 6^-} |x-6| = |6-6| = |0| = 0$

g) $\lim_{x \rightarrow 6} |x-6| = 0$ because $\lim_{x \rightarrow 6^+} = \lim_{x \rightarrow 6^-}$

h) $\lim_{x \rightarrow 0^+} \frac{|x|}{x}$ approaching from \leftarrow so use $\frac{|0.001|}{0.001} = 1$

i) $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$ approaching from \rightarrow so use $\frac{|-0.001|}{-0.001} = \frac{0.001}{-0.001} = -1$

j) $\lim_{x \rightarrow 0} \frac{|x|}{x}$ does not exist because $\lim_{x \rightarrow 0^-} \neq \lim_{x \rightarrow 0^+}$

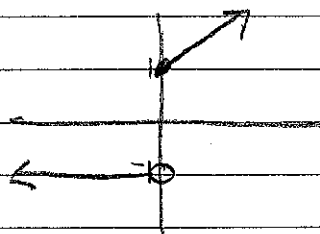
2. $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$

0 is possible discontinuity

a) $\lim_{x \rightarrow 0^-} f(x) = -1$

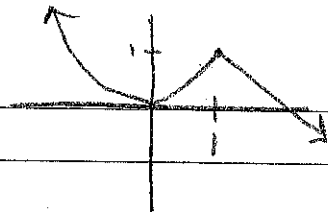
b) $\lim_{x \rightarrow 0^+} f(x) = x+1 = 0+1 = 1$

c) $\lim_{x \rightarrow 0} f(x)$ does not exist because $-1 \neq 1$



2-b cont.

$$3. g(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2-x & \text{if } x > 1 \end{cases}$$



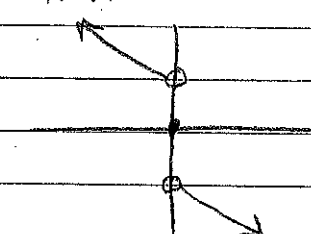
a) $\lim_{x \rightarrow 1^-} g(x) = 1^2 = 1$

b) $\lim_{x \rightarrow 1^+} g(x) = 2 - 1 = 1$

c) $\lim_{x \rightarrow 1} g(x) = 1$ because $\lim_{x \rightarrow 1^-} = \lim_{x \rightarrow 1^+}$

4.

$$h(x) = \begin{cases} 1-x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ -x-1 & \text{if } x > 0 \end{cases}$$



a) $\lim_{x \rightarrow 0^-} h(x) = 1 - 0 = 1$

b) $\lim_{x \rightarrow 0^+} h(x) = -0 - 1 = -1$

c) $\lim_{x \rightarrow 0} h(x)$ does not exist because $1 \neq -1$

5.

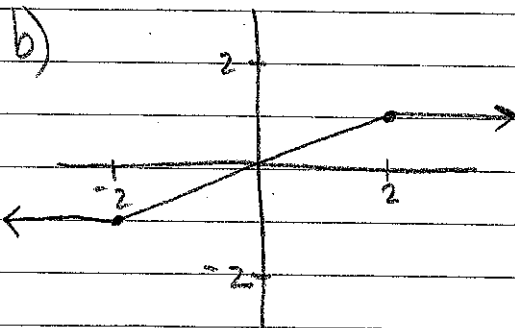
$$f(x) = \begin{cases} -1 & \text{if } x \leq -2 \\ \frac{1}{2}x & \text{if } -2 < x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

a) $\lim_{x \rightarrow -2^-} f(x) = -1$

ii) $\lim_{x \rightarrow -2^+} f(x) = \frac{1}{2}(-2) = -1$

iii) $\lim_{x \rightarrow 2^-} f(x) = \frac{1}{2}(2) = 1$

iv) $\lim_{x \rightarrow 2^+} f(x) = 1$



c) The graph is continuous everywhere