Calculus 2-8

1. a) $y = 2x - x^2$ at point $(2,0)$

i) $m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2} \frac{2x - x^2 - [2(2) - 2^2]}{x - 2}$

$= \lim_{x \to 2} \frac{2x - x^2 - (\cancel{4 - 4})}{x - 2}$

$= \lim_{x \to 2} \frac{2x - x^2}{x - 2}$

$= \lim_{x \to 2} \frac{x(\cancel{2 - x})(-1)}{\cancel{x - 2}}$

$= \lim_{x \to 2} -x$

$= -2$

ii) $m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \frac{2(2+h) - (2+h)^2 - (2 \cdot 2 - 2^2)}{h}$

$= \lim_{h \to 0} \frac{4 + 2h - (4 + 4h + h^2) - (\cancel{4 - 4})}{h}$

$= \lim_{h \to 0} \frac{\cancel{4} + 2h - \cancel{4} - 4h - h^2}{h}$

$= \lim_{h \to 0} \frac{-2h - h^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(-2 - h)}{\cancel{h}}$

$= \lim_{h \to 0} -2 - h$

$= -2 - 0$

$= -2$

2-8 cont.

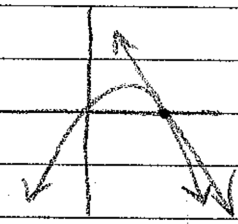
1. cont.

b) $y - y_1 = m(x - x_1)$

$y - 0 = -2(x - 2)$

$y = -2x + 4$

c)



2. a) $y = x^3$ at point $(1,1)$

i) $m = \lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{x^3 - 1^3}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{x-1}}$

$= \lim\limits_{x \to 1} x^2 + x + 1$

$= 1^2 + 1 + 1$

$= 3$

ii) $m = \lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

$(1+h)(1+h)(1+h) =$

$(1 + 2h + h^2)(1+h) =$

$1 + 2h + h^2 + h + 2h^2 + h^3 =$

$1 + 3h + 3h^2 + h^3$

$= \lim\limits_{h \to 0} \dfrac{(1+h)^3 - 1^3}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{1} + 3h + 3h^2 + h^3 - \cancel{1}}{h}$

$= \lim\limits_{h \to 0} \dfrac{3h + 3h^2 + h^3}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(3 + 3h + h^2)}{\cancel{h}}$

$= \lim\limits_{h \to 0} 3 + 3h + h^2$

$= 3 + 3 \cdot 0 + 0^2$

$= 3$

2-8 cont.

2. cont.

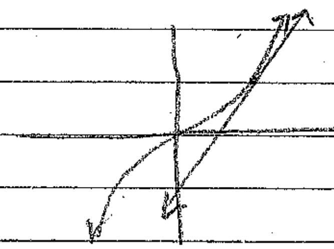
b) $y - y_1 = m(x - x_1)$

$y - 1 = 3(x-1)$

$y - 1 = 3x - 3$

$y = 3x - 2$

c)



3. a) $f(x) = 4 - x + 3x^2$ at point $(-1, 8)$

$f(-1) = 8$

$$m = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h \to 0} \frac{4 - (-1+h) + 3(1+h)^2 - 8}{h}$$

$$= \lim_{h \to 0} \frac{4 + 1 - h + 3(1 + 2h + h^2) - 8}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5} - h + \cancel{3} - 6h + 3h^2 \cancel{-8}}{h}$$

$$= \lim_{h \to 0} \frac{-7h + 3h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-7 + 3h)}{\cancel{h}}$$

$$= \lim_{h \to 0} -7 + 3h$$

$$= -7 + 3 \cdot 0$$

$$= -7$$

$y - 8 = -7(x - -1)$

$y - 8 = -7x - 7$

$y = -7x + 1$

2-8 cont.

3) b) $f(x) = x^3 - x$ at point $(0,0)$

$$m = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(0+h)^3 - (0+h) - \boxed{0^3 - 0}}{h}$$

$$= \lim_{h \to 0} \frac{h^3 - h - 0}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h^2 - 1)}{\cancel{h}}$$

$$= \lim_{h \to 0} h^2 - 1$$

$$= 0^2 - 1$$

$$= -1$$

$$y - 0 = -1(x - 0)$$

$$y = -x$$

c) $g(x) = \frac{2x+1}{x-1}$ at point $(2,5)$

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2(2+h)+1}{2+h-1} - \boxed{\frac{2(2)+1}{2-1}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4+2h+1}{h+1} - \frac{5}{1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2h+5}{h+1} - 5}{h} \cdot \frac{(h+1)}{(h+1)}$$

$$= \lim_{h \to 0} \frac{2h+5-5(h+1)}{h(h+1)}$$

$$= \lim_{h \to 0} \frac{2h+5-5h-5}{h(h+1)}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(h+1)}$$

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2-8 cont.

3. c) cont.

$$m = \lim_{h \to 0} \frac{-3}{h+1}$$

$$= \frac{-3}{0+1}$$

$$= -3$$

$$y - 5 = -3(x-2)$$

$$y - 5 = -3x + 6$$

$$y = -3x + 11$$

d) $g(x) = \frac{1}{\sqrt{x}}$ at point $(1,1)$

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{1+h}} - \frac{1}{\sqrt{1}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{1+h}} - 1}{h} \cdot \frac{\sqrt{1+h}}{\sqrt{1+h}}$$

$$= \lim_{h \to 0} \frac{1 - \sqrt{1+h}}{h(\sqrt{1+h})} \cdot \frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}$$

$$= \lim_{h \to 0} \frac{1-(1+h)}{h(\sqrt{1+h} + 1 + h)}$$

$$= \lim_{h \to 0} \frac{-1\cancel{h}}{\cancel{h}(\sqrt{1+h} + 1 + h)}$$

$$= \frac{-1}{\sqrt{1+0} + 1 + 0}$$

$$= \frac{-1}{2}$$

$$y - 1 = -\frac{1}{2}(x-1)$$

$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$

$$2y - 2 = -x + 1$$

$$x + 2y - 3 = 0$$