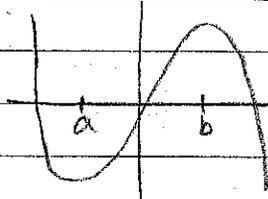
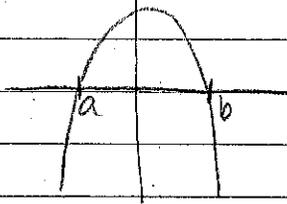


Calculus 5-3

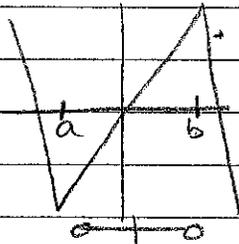
1. a)



zero at a and b
pos slope between a and b
neg slope left of a and right of b

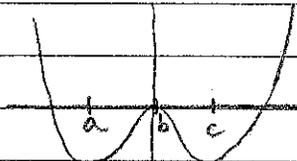


b)

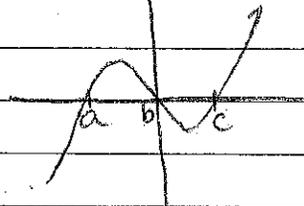


no tangent at a or b
pos slope between a and b
neg slope left of a and right of b

c)



zero at a , b , and c
pos slope between a and b
and right of c
neg slope between b and c
and left of a



5-3 cont.

2. A function is not differentiable at any sharp point on the graph

a) -3, 0, 2, 4

b) -6, -4, -2, 0, 2, 4, 6

3. a) $f(x) = 7x - x^2$

$$f'(a) = \lim_{h \rightarrow 0} \frac{7(a+h) - (a+h)^2 - (7a - a^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{7a + 7h - (a^2 + 2ah + h^2) - 7a + a^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{7h - a^2 - 2ah - h^2 - a^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{7h - 2ah - h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(7 - 2a - h)}{h}$$

$$= \lim_{h \rightarrow 0} 7 - 2a - h$$

$$= 7 - 2a - 0$$

$$= 7 - 2a$$

b) $f(x) = 2x^3 + 5$

$$f'(a) = \lim_{h \rightarrow 0} \frac{2(a+h)^3 + 5 - (2a^3 + 5)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(a^3 + 3a^2h + 3ah^2 + h^3) + 5 - 2a^3 - 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2a^3 + 6a^2h + 6ah^2 + 2h^3 - 2a^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(6a^2 + 6ah + 2h^2)}{h}$$

$$= 6a^2 + 6a(0) + 2(0)^2$$

$$= 6a^2$$

5-3 cont.

3. c) $f(x) = \frac{1+2x}{1+x}$

$$f'(a) = \lim_{h \rightarrow 0} \frac{1+2(a+h)}{1+(a+h)} \cdot \frac{1+2a}{1+a}$$

$$= \lim_{h \rightarrow 0} \frac{[1+2(a+h)](1+a) - (1+2a)(1+a+h)}{[1+(a+h)](1+a)} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(1+2a+2h)(1+a) - (1+a+h+2a+2a^2+2ah)}{h(1+a+h)(1+a)}$$

$$= \lim_{h \rightarrow 0} \frac{1+a+2a+2a^2+2h+2ah - (1+3a+h+2a^2+2ah)}{h(1+a+h)(1+a)}$$

$$= \lim_{h \rightarrow 0} \frac{1+3a+2a^2+2h+2ah - 1-3a-h-2a^2-2ah}{h(1+a+h)(1+a)}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(1+a+h)(1+a)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{(1+a+h)(1+a)}$$

$$= \frac{1}{(1+a+0)(1+a)}$$

$$= \frac{1}{(1+a)^2}$$

d) $f(x) = \sqrt{x}$

$$f'(a) = \lim_{h \rightarrow 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}}$$

$$= \lim_{h \rightarrow 0} \frac{a+h-a}{h(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{a+h} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{a+0} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$

5-3 cont.

4. a) $f(x) = 3x^2 + 2x - 4$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h)^2 + 2(x+h) - 4 - (3x^2 + 2x - 4)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) + 2x + 2h - 4 - 3x^2 - 2x + 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2h - 3x^2 - 2x + 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$= \lim_{h \rightarrow 0} 6x + 3h + 2$$

$$= 6x + 3 \cdot 0 + 2$$

$$= 6x + 2$$

b) $f(x) = x^2 - x^3$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x+h)^3 - (x^2 - x^3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - (x^3 + 3x^2h + 3xh^2 + h^3) - x^2 + x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3x^2h - 3xh^2 - h^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x + h - 3x^2 - 3xh - h^2}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h - 3x^2 - 3xh - h^2$$

$$= 2x + 0 - 3x^2 - 3x \cdot 0 - 0^2$$

$$= 2x - 3x^2$$

5-3 cont.

4. c) $f(x) = x^4$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$

$$= \lim_{h \rightarrow 0} 4x^3 + 6x^2h + 4xh^2 + h^3$$

$$= 4x^3 + 6x^2 \cdot 0 + 4x \cdot 0^2 + 0^3$$

$$= 4x^3$$

d) $f(x) = \frac{x}{5x-1}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x+h}{5(x+h)-1} \cdot \frac{x}{5x-1}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)(5x-1) - x[5(x+h)-1]}{(5x+5h-1)(5x-1)} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5x^2 - x + 5xh - h - x(5x+h-1)}{h(5x+5h-1)(5x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{5x^2 - x + 5xh - h - 5x^2 - 5hx + x}{h(5x+5h-1)(5x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h(5x+5h-1)(5x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{(5x+5h-1)(5x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{(5x+5h-1)(5x-1)}$$

$$= \frac{-1}{(5x+5 \cdot 0 - 1)(5x-1)}$$

$$= \frac{-1}{(5x-1)^2}$$

5-3 cont.

5 a) $f(x) = \sqrt{2x-1}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)-1} - \sqrt{2x-1}}{h} \cdot \frac{\sqrt{2(x+h)-1} + \sqrt{2x-1}}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}$$

$$= \lim_{h \rightarrow 0} \frac{2(x+h)-1 - (2x-1)}{h(\sqrt{2(x+h)-1} + \sqrt{2x-1})}$$

$$= \lim_{h \rightarrow 0} \frac{2x+2h-1-2x+1}{h(\sqrt{2(x+h)-1} + \sqrt{2x-1})}$$

$$= \lim_{h \rightarrow 0} \frac{2}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}$$

$$= \frac{2}{\sqrt{2(x+0)-1} + \sqrt{2x-1}}$$

$$= \frac{2}{\sqrt{2x-1} + \sqrt{2x-1}}$$

$$= \frac{2}{2\sqrt{2x-1}}$$

$$= \frac{1}{\sqrt{2x-1}}$$

Domain:

for $f(x)$: $2x-1 \geq 0$

$$2x \geq 1$$

$$x \geq \frac{1}{2}$$

$$\{x \mid x \geq \frac{1}{2}\}$$

for $f'(x)$: $2x-1 > 0$

$$2x > 1$$

$$x > \frac{1}{2}$$

cannot = 0 because
it is in the
denominator

$$\{x \mid x > \frac{1}{2}\}$$

5-3 cont.

$$5. b) g(x) = \frac{1}{\sqrt{x}}$$

$$g'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} \cdot \frac{1}{h}}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h \rightarrow 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \rightarrow 0} \frac{x - x - h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x+0}(\sqrt{x} + \sqrt{x+0})}$$

$$= \frac{-1}{x \cdot 2\sqrt{x}}$$

they have $\frac{-1}{2\sqrt{x^3}}$ which is the same.

Domain:

$$g(x): x > 0$$

$$\{x | x > 0\}$$

$$g'(x): x^3 > 0$$

$$x > 0$$

$$\{x | x > 0\}$$

5-3: cont.

5. c) $F(x) = \frac{3-2x}{4+x}$

$$F'(x) = \lim_{h \rightarrow 0} \frac{\frac{3-2(x+h)}{4+x+h} - \frac{3-2x}{4+x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(3-2x-2h)(4+x) - (3-2x)(4+x+h)}{h(4+x+h)(4+x)}$$

$$= \lim_{h \rightarrow 0} \frac{12+3x-8x-2x^2-8h-2hx - (12+3x+3h-8x-2x^2-2xh)}{h(4+x+h)(4+x)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{12} - 5x - 2x^2 - 8h - 2hx - \cancel{12} + 5x - 3h + 2x^2 + 2xh}{h(4+x+h)(4+x)}$$

$$= \lim_{h \rightarrow 0} \frac{-11h}{h(4+x+h)(4+x)}$$

$$= \frac{-11}{(4+x+0)(4+x)}$$

$$= \frac{-11}{(4+x)^2}$$

Domain:

$$F(x): \begin{aligned} 4+x &\neq 0 \\ x &\neq -4 \end{aligned}$$

$$F'(x): \begin{aligned} 4+x &\neq 0 \\ x &\neq -4 \end{aligned}$$

$$\{x \mid x \neq -4\}$$

$$\{x \mid x \neq -4\}$$

5-3 cont.

$$5. d) f(t) = \frac{2}{t^2-1}$$

$$f'(t) = \lim_{h \rightarrow 0} \frac{2}{(t+h)^2-1} - \frac{2}{t^2-1}$$

$$= \lim_{h \rightarrow 0} \frac{2(t^2-1) - 2[(t+h)^2-1]}{h[(t+h)^2-1](t^2-1)} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2t^2 - 2 - 2(t^2 + 2th + h^2 - 1)}{h[(t+h)^2-1](t^2-1)}$$

$$= \lim_{h \rightarrow 0} \frac{2t^2 - 2 - 2t^2 - 4th - 2h^2 + 2}{h[(t+h)^2-1](t^2-1)}$$

$$= \lim_{h \rightarrow 0} \frac{h(-4t-2h)}{h[(t+h)^2-1](t^2-1)}$$

$$= \frac{-4t - 2(0)}{[(t+0)^2-1](t^2-1)}$$

$$= \frac{-4t}{(t^2-1)^2}$$

Domain:

$$f(t): t^2 - 1 \neq 0$$

$$t^2 \neq 1$$

$$t \neq \pm 1$$

$$\{t \mid t \neq \pm 1\}$$

$$f'(t): t^2 - 1 \neq 0$$

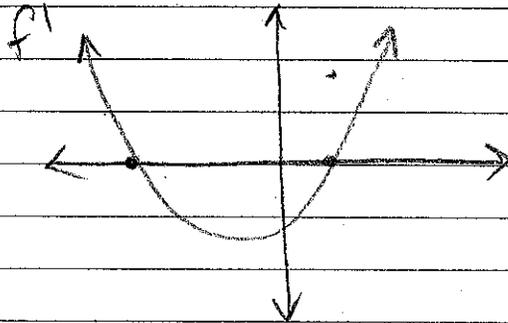
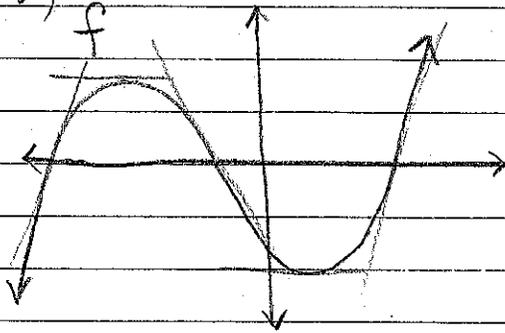
$$t^2 \neq 1$$

$$t \neq \pm 1$$

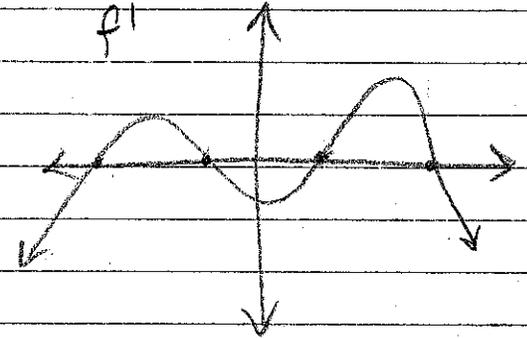
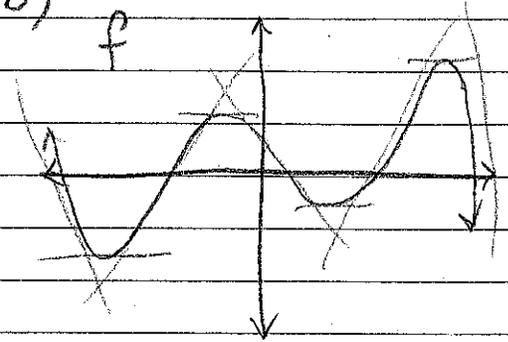
$$\{t \mid t \neq \pm 1\}$$

5-3 cont.

b. a)



b)



- if the tangent is horizontal - f' is at zero

- if the slope of the tangent is positive - f' is above the x-axis

- if the slope of the tangent is negative - f' is below the x-axis