

# Calculus 6-1

1. a)  $f(x) = 32$   
 $f'(x) = 0$

b)  $f(x) = x^4$   
 $f'(x) = 4x^{4-1}$   
 $= 4x^3$

c)  $y = x^{12}$   
 $y' = 12x^{12-1}$   
 $= 12x^{11}$

d)  $y = -3.724$   
 $y' = 0$

e)  $f(x) = x$   
 $f'(x) = 1x^0$   
 $= 1 \cdot 1$   
 $= 1$

f)  $f(x) = x^\pi$   
 $f'(x) = \pi x^{\pi-1}$

g)  $f(x) = x^{43}$   
 $f'(x) = 43x^{43-1}$   
 $= 43x^{42}$

h)  $f(x) = 2^5 \leftarrow \text{constant}$   
 $f'(x) = 0$

i)  $g(x) = x^{-2}$   
 $g'(x) = -2x^{-2-1}$   
 $= -2x^{-3}$

j)  $g(x) = x^{\frac{3}{2}}$   
 $g'(x) = \frac{3}{2}x^{\frac{3}{2}-1}$   
 $= \frac{3}{2}x^{\frac{1}{2}}$  or  $\frac{3\sqrt{x}}{2}$

*Handwritten notes in a cloud:*  
 $\frac{3}{2} - 1 = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

2. a)  $f(x) = 8x^{12}$   
 $f'(x) = 8 \cdot 12x^{12-1}$   
 $= 96x^{11}$

b)  $f(x) = -3x^9$   
 $f'(x) = -3 \cdot 9x^{9-1}$   
 $= -27x^8$

c)  $f(t) = 3t^{\frac{4}{3}}$   
 $f'(t) = 3 \cdot \frac{4}{3}t^{\frac{4}{3}-1}$   
 $= 4t^{\frac{1}{3}}$

*Handwritten notes in a cloud:*  
 $\frac{4}{3} - 1 = \frac{4}{3} - \frac{3}{3} = \frac{1}{3}$

d)  $g(t) = 8t^{-\frac{3}{4}}$   
 $g'(t) = 8 \cdot \left(-\frac{3}{4}\right)t^{-\frac{3}{4}-1}$   
 $= -6t^{-\frac{7}{4}}$

*Handwritten notes in a cloud:*  
 $-\frac{3}{4} - 1 = -\frac{3}{4} - \frac{4}{4} = -\frac{7}{4}$

6-1 cont.

2. e)  $y = \frac{1}{x^4}$

$$\begin{aligned} &= x^{-4} \\ y' &= -4x^{-4-1} \\ &= -4x^{-5} \\ &= \frac{-4}{x^5} \end{aligned}$$

f)  $y = \frac{2}{x^2}$

$$\begin{aligned} &= 2x^{-2} \\ y' &= 2(-2)x^{-2-1} \\ &= -4x^{-3} \\ &= \frac{-4}{x^3} \end{aligned}$$

g)  $g(t) = (2t)^3$

$$\begin{aligned} &= 8t^3 \\ g'(t) &= 8 \cdot 3t^{3-1} \\ &= 24t^2 \end{aligned}$$

h)  $h(y) = \left(\frac{y}{3}\right)^2 = \frac{y^2}{9} = \frac{1}{9}y^2$

$$\begin{aligned} h'(y) &= \frac{1}{9} \cdot 2y^{2-1} \\ &= \frac{2}{9}y \end{aligned}$$

i)  $f(x) = \sqrt[3]{x}$

$$\begin{aligned} &= x^{\frac{1}{3}} \\ f'(x) &= \frac{1}{3}x^{\frac{1}{3}-1} \\ &= \frac{1}{3}x^{-\frac{2}{3}} \end{aligned}$$

j)  $f(x) = \sqrt[5]{x^2}$

$$\begin{aligned} &= x^{\frac{2}{5}} \\ f'(x) &= \frac{2}{5}x^{\frac{2}{5}-1} \\ &= \frac{2}{5}x^{-\frac{3}{5}} \end{aligned}$$

k)  $y = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$$\begin{aligned} y' &= -\frac{1}{2}x^{-\frac{1}{2}-1} \\ &= -\frac{1}{2}x^{-\frac{3}{2}} \end{aligned}$$

l)  $y = \frac{3}{\sqrt[4]{x}} = 3x^{-\frac{1}{4}}$

$$\begin{aligned} y' &= 3 \cdot \frac{1}{4}x^{-\frac{1}{4}-1} \\ &= \frac{3}{4}x^{-\frac{5}{4}} \end{aligned}$$

m)  $y = \sqrt{3}x^{\frac{1}{12}}$

$$\begin{aligned} y' &= \sqrt{3} \cdot \frac{1}{12}x^{\frac{1}{12}-1} \\ &= \frac{\sqrt{3}}{12}x^{-\frac{11}{12}} \end{aligned}$$

n)  $y = (x^3)^4$

$$\begin{aligned} &= x^{12} \\ y' &= 12x^{12-1} \\ &= 12x^{11} \end{aligned}$$

6-1 cont.

3. a)  $f(x) = 2x^3, X = \frac{1}{3}$

$$f'(x) = 6x^2$$

$$f'(\frac{1}{3}) = 6(\frac{1}{3})^2$$

$$= 6(\frac{1}{9})$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

b)  $f(x) = X^{1.4}, X=1$

$$f'(x) = 1.4 X^{0.4}$$

$$f'(1) = 1.4(1)^{0.4}$$

$$= 1.4(1)$$

$$= 1.4$$

c)  $g(x) = x^{-3}, x = -1$

$$g'(x) = -3x^{-4}$$

$$g'(-1) = -3(-1)^{-4}$$

$$= -3$$

$$\frac{-3}{(-1)^4}$$

$$= \frac{-3}{1}$$

$$= -3$$

d)  $g(x) = \sqrt[5]{x}, X=32$

$$= X^{\frac{1}{5}}$$

$$g'(x) = \frac{1}{5} X^{-\frac{4}{5}}$$

$$= \frac{1}{5} \cdot \frac{1}{X^{\frac{4}{5}}}$$

$$= \frac{1}{5 \sqrt[5]{X^4}}$$

$$= \frac{1}{5 \sqrt[5]{32^4}}$$

$$g'(32) = \frac{1}{5 \sqrt[5]{32^4}}$$

$$= \frac{1}{5 \cdot 16}$$

$$= \frac{1}{80}$$

e)  $y = \sqrt{x^3}, X=8$

$$= X^{\frac{3}{2}}$$

$$y' = \frac{3}{2} X^{\frac{1}{2}}$$

$$= \frac{3\sqrt{x}}{2}$$

$x=8$

$$y' = \frac{3\sqrt{8}}{2}$$

$$= \frac{3 \cdot 2\sqrt{2}}{2}$$

$$= 3\sqrt{2}$$

f)  $y = \frac{6}{x}, X=-3$

$$= 6x^{-1}$$

$$y' = -6x^{-2}$$

$$= \frac{-6}{x^2}$$

$x=-3$   $y' = \frac{-6}{(-3)^2}$

$$= \frac{-6}{9}$$

$$= -\frac{2}{3}$$

$$= -\frac{2}{3}$$

6-1 cont.

4. a)  $y = x^5, (2, 32)$

$$y' = 5x^4$$

$$x=2 \rightarrow y' = 5(2)^4$$

$$= 5 \cdot 16$$

$$= 80$$

$$y - 32 = 80(x - 2)$$

$$y - 32 = 80x - 160$$

$$y = 80x - 128$$

b)  $y = 2\sqrt{x}, (9, 6)$

$$= 2x^{\frac{1}{2}}$$

$$y' = 2 \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{x}}$$

$$x=9 \rightarrow y' = \frac{1}{\sqrt{9}}$$

$$= \frac{1}{3}$$

$$y - 6 = \frac{1}{3}(x - 9)$$

$$3(y - 6) = 3 \cdot \frac{1}{3}(x - 9)$$

$$3y - 18 = x - 9$$

$$0 = x - 3y + 9$$

c)  $xy = 1, (5, \frac{1}{5})$

$$y = \frac{1}{x}$$

$$= x^{-1}$$

$$y' = -1x^{-2}$$

$$= \frac{-1}{x^2}$$

$$x=5 \rightarrow y' = \frac{-1}{5^2}$$

$$= \frac{-1}{25}$$

$$y - \frac{1}{5} = \frac{-1}{25}(x - 5)$$

$$25(y - \frac{1}{5}) = 25(\frac{-1}{25})(x - 5)$$

$$25y - 5 = -1(x - 5)$$

$$25y - 5 = -x + 5$$

$$x + 25y - 10 = 0$$

d)  $y = \sqrt[3]{x}, (-8, -2)$

$$y = x^{\frac{1}{3}}$$

$$y' = \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{1}{3x^{\frac{2}{3}}}$$

$$x = -8 \rightarrow y' = \frac{1}{3(-8)^{\frac{2}{3}}}$$

$$= \frac{1}{3\sqrt[3]{-8^2}}$$

$$= \frac{1}{3(-2)^2}$$

$$= \frac{1}{3(4)}$$

$$= \frac{1}{12}$$

$$y - -2 = \frac{1}{12}(x - -8)$$

$$y + 2 = \frac{1}{12}(x + 8)$$

$$12(y + 2) = 12(\frac{1}{12})(x + 8)$$

$$12y + 24 = x + 8$$

$$0 = x - 12y - 16$$

6-1 cont.

5.  $y = 3x^2$  slope = 24

$y' = 6x \rightarrow$  slope

$$\frac{24}{6} = \frac{6x}{6}$$

$$4 = x$$

$\rightarrow y = 3(4)^2$   
 $= 3 \cdot 16$   
 $= 48$

slope = 24 at (4, 48)

6.  $y = x - \sqrt{x}$  parallel to  $6x - y = 4$

$$= x \cdot x^{-\frac{1}{2}}$$

$$= x^{\frac{1}{2}}$$

$$y' = \frac{3}{2}x^{-\frac{1}{2}}$$

$$= \frac{3\sqrt{x}}{2} \rightarrow \text{slope } \frac{2 \cdot 3\sqrt{x}}{2} = 6 \cdot 2$$

$$\frac{3\sqrt{x}}{3} = \frac{12}{3}$$

$$\sqrt{x} = 4^2$$

$$x = 16$$

$\rightarrow y = 16 - \sqrt{16}$   
 $= 16 - 4$   
 $= 12$

tangent is parallel at (16, 12)

7.  $y = -2x^4$  perpendicular to  $x - y + 1 = 0$

$y' = -8x^3 \rightarrow$  slope.

$x + 1 = y$  slope = 1

perpendicular slope = -1

$$-8x^3 = -1$$

$$x^3 = \frac{1}{8}$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$\rightarrow y = -2\left(\frac{1}{2}\right)^4$   
 $= -2\left(\frac{1}{16}\right)$

$$= \frac{-2}{16}$$

$$= -\frac{1}{8}$$

tangent is perpendicular at  $\left(\frac{1}{2}, -\frac{1}{8}\right)$