

# Calculus 6-2

1. a)  $f(x) = x^2 + 4x$   
 $f'(x) = 2x^{2-1} + 4$   
 $= 2x + 4$

b)  $f(x) = 3x^5 - 6x^4 + 2$   
 $f'(x) = 3 \cdot 5x^{5-1} - 6 \cdot 4x^{4-1} + 0$   
 $= 15x^4 - 24x^3$

c)  $g(x) = x^{10} + 25x^5 - 50$   
 $g'(x) = 10x^9 + 125x^4$

d)  $g(x) = x^2 - \frac{2}{x^2}$   
 $= x^2 - 2x^{-2}$   
 $g'(x) = 2x - (2)(-2)x^{-3}$   
 $= 2x + 4x^{-3}$   
 $= 2x + \frac{4}{x^3}$

e)  $h(x) = \sqrt{x} - 5x^4$   
 $= x^{\frac{1}{2}} - 5x^4$   
 $h'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 20x^3$   
 $= \frac{1}{2\sqrt{x}} - 20x^3$

f)  $h(x) = (x-1)(x+6)$   
 $= x^2 + 6x - x - 6$   
 $= x^2 + 5x - 6$   
 $h'(x) = 2x + 5$

g)  $y = \frac{x+1}{\sqrt{x}}$   
 $= \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}$   
 $= x \cdot x^{-\frac{1}{2}} + x^{-\frac{1}{2}}$   
 $= x^{\frac{1}{2}} + x^{-\frac{1}{2}}$   
 $y' = \frac{1}{2}x^{-\frac{1}{2}} + (-\frac{1}{2})x^{-\frac{3}{2}}$   
 $= \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

h)  $y = t^5 - 6t^{-5}$   
 $y' = 5t^4 - 6(-5)t^{-6}$   
 $= 5t^4 + 30t^{-6}$

j)  $F(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}$   
 $= x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^{\frac{1}{4}}$   
 $F'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{4}}$   
 $= \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{3x^{\frac{2}{3}}} + \frac{1}{4x^{\frac{3}{4}}}$

i)  $f(t) = (1+t)^3$   
 $= (1+t)(1+t)(1+t)$   
 $= (1+2t+t^2)(1+t)$   
 $= 1+t+2t+2t^2+t^2+t^3$   
 $= 1+3t+3t^2+t^3$   
 $f'(t) = 3+6t+3t^2$

6-2 cont.

1. cont.

$$k) u(t) = a + \frac{b}{t} + \frac{c}{t^2}$$

$$= a + bt^{-1} + ct^{-2}$$

$$u'(t) = -bt^{-2} - 2ct^{-3}$$

$$= -\frac{b}{t^2} - \frac{2c}{t^3}$$

$$l) v(r) = \sqrt{r}(2+3r)$$

$$= 2\sqrt{r} + 3r\sqrt{r}$$

$$= 2r^{\frac{1}{2}} + 3r^{\frac{3}{2}}$$

$$= 2r^{\frac{1}{2}} + 3r^{\frac{3}{2}}$$

$$v'(r) = 2 \cdot \frac{1}{2} r^{-\frac{1}{2}} + 3 \cdot \frac{3}{2} r^{\frac{1}{2}}$$

$$= \frac{1}{r^{\frac{1}{2}}} + \frac{9}{2} r^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{r}} + \frac{9\sqrt{r}}{2}$$

$$2. a) f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$$

$$f'(x) = 1 + \frac{1}{2} \cdot 2x + \frac{1}{3} \cdot 3x^2 + \frac{1}{4} \cdot 4x^3$$

$$= 1 + x + x^2 + x^3$$

domain for both =  $\{x | x \in \mathbb{R}\}$

$$b) f(x) = 4x - \sqrt[4]{x} \rightarrow \text{domain: } \{x | x \geq 0\}$$

$$= 4x - x^{\frac{1}{4}}$$

$$f'(x) = 4 - \frac{1}{4}x^{-\frac{3}{4}} \rightarrow \text{domain: } \{x | x > 0\}$$

$$= 4 - \frac{1}{4x^{\frac{3}{4}}}$$

$$c) f(x) = x + \frac{\sqrt{10}}{x^5} \rightarrow \text{domain: } \{x | x \neq 0\}$$

$$= x + \sqrt{10}x^{-5}$$

$$f'(x) = 1 + \sqrt{10}(-5)x^{-6} \rightarrow \text{domain } \{x | x \neq 0\}$$

$$= 1 - \frac{5\sqrt{10}}{x^6}$$

16-2 cont.

2. cont.

$$d) f(x) = \frac{\sqrt{x+2}}{\sqrt{x}} \rightarrow \text{domain: } \{x | x > 0\}$$

$$= x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$$
$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 2\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} \rightarrow \text{domain: } \{x | x > 0\}$$

$$= \frac{1}{2\sqrt{x}} - \frac{1}{x^{\frac{3}{2}}}$$

3. a)  $y = x^3 - x^2 + x - 1, (1, 0)$

$$y' = 3x^2 - 2x + 1 \rightarrow \text{slope}$$

$$y' \text{ at } x=1: 3(1)^2 - 2(1) + 1$$

$$3 - 2 + 1$$

2  $\rightarrow$  slope at  $x=1$

$$y - 0 = 2(x - 1)$$

$$y = 2x - 2$$

b)  $y = 7\sqrt{x} - 3x, (1, 4)$

$$= 7x^{\frac{1}{2}} - 3x$$

$$y' = \frac{7 \cdot \frac{1}{2} x^{-\frac{1}{2}}}{2} - 3$$

$$= \frac{7}{2\sqrt{x}} - 3$$

$$\text{at } x=1: \frac{7}{2\sqrt{1}} - 3$$

$$\frac{7}{2} - 3$$

$$0.5$$

$$y - 4 = 0.5(x - 1)$$
$$2(y - 4) = (0.5x - 0.5) \cdot 2$$

$$2y - 8 = x - 1$$

$$0 = x - 2y + 7$$

c)  $y = x + \frac{6}{x}, (2, 5)$

$$= x + 6x^{-1}$$

$$y' = 1 + 6(-1)x^{-2}$$

$$= 1 - \frac{6}{x^2}$$

$$1 - \frac{6}{2^2}$$

$$1 - \frac{6}{4}$$

$$-0.5$$

$$y - 5 = -0.5(x - 2)$$

$$2(y - 5) = (-0.5x + 1) \cdot 2$$

$$2y - 10 = -x + 2$$

$$x + 2y - 12 = 0$$

6-2 cont.

3. d)  $y = (x^2 + 1)^2, (-1, 4)$

$$y = (x^2 + 1)(x^2 + 1)$$

$$y = x^4 + 2x^2 + 1$$

$$y' = 4x^3 + 4x$$

$$4(-1)^3 + 4(-1)$$

$$-4 - 4$$

$$-8$$

$$y - 4 = -8(x + 1)$$

$$y - 4 = -8x - 8$$

$$8x + y + 4 = 0$$

4.  $h = 40t - 5t^2$  after 2s, 4s, 5s

$$h' = 40 - 10t$$

2sec:  $40 - 10 \cdot 2$

$$40 - 20$$

$$20 \text{ m}$$

4sec:  $40 - 10 \cdot 4$

$$40 - 40$$

$$0 \text{ m}$$

5sec:  $40 - 10 \cdot 5$

$$40 - 50$$

$$-10 \text{ m}$$