

# Calculus 6-3

1. a)  $f(x) = (2x-1)(x^2+1)$

$$f'(x) = (2x-1)(2x) + (x^2+1)(2)$$

b)  $f(x) = x(3x-8)$

$$f'(x) = x(3) + (3x-8)(1)$$

c)  $y = x^2(1+x-3x^2)$

$$y' = x^2(1+6x) + (1+x-3x^2)(2x)$$

d)  $y = (x^3+x^2+1)(x^2+2)$

$$y' = (x^3+x^2+1)(2x) + (x^2+2)(3x^2+2x)$$

e)  $f(t) = (t^4+t^2-1)(t^2-2)$

$$f'(t) = (t^4+t^2-1)(2t) + (t^2-2)(4t^3+2t)$$

f)  $f(t) = \sqrt[3]{t}(1-t)$

$$= t^{\frac{1}{3}}(1-t)$$

$$f'(t) = t^{\frac{1}{3}}(-1) + (1-t)\left(\frac{1}{3}t^{-\frac{2}{3}}\right)$$

g)  $F(y) = \sqrt{y}(y-2\sqrt{y}+2)$

$$= y^{\frac{1}{2}}(y-2y^{\frac{1}{2}}+2)$$

$$F'(y) = y^{\frac{1}{2}}(1-y^{\frac{1}{2}}) + (y-2y^{\frac{1}{2}}+2)\left(\frac{1}{2}y^{-\frac{1}{2}}\right)$$

h)  $G(y) = (y-y^2)(2y-y^{\frac{4}{3}})$

$$G'(y) = (y-y^2)\left(2-\frac{4}{3}y^{\frac{1}{3}}\right) + (2y-y^{\frac{4}{3}})(1-2y)$$

2. a)  $y = x^3(x^2+2x+3)$

$$y' = x^3(2x+2) + (x^2+2x+3)(3x^2)$$

$$= 2x^4+2x^3+3x^4+6x^3+9x^2$$

$$= 5x^4+8x^3+9x^2$$

6-3 cont.

2. cont.

$$b) y = x^{-2}(x^3 - 3x^2 + 6)$$

$$\begin{aligned} y' &= x^{-2}(3x^2 - 6x) + (x^3 - 3x^2 + 6)(-2x^{-3}) \\ &= 3x^0 - 6x^{-1} - 2x^0 + 6x^{-1} - 12x^{-3} \\ &= 1 - 12x^{-3} \end{aligned}$$

$$c) f(x) = (1-x^2)(2-x^3)$$

$$\begin{aligned} f'(x) &= (1-x^2)(-3x^2) + (2-x^3)(-2x) \\ &= -3x^2 + 3x^4 - 4x + 2x^4 \\ &= 5x^4 + 3x^2 - 4x \end{aligned}$$

$$d) f(x) = (3x^3 + 4)(1 - 2x^3)$$

$$\begin{aligned} f'(x) &= (3x^3 + 4)(-6x^2) + (1 - 2x^3)(9x^2) \\ &= -18x^5 - 24x^2 + 9x^2 - 18x^5 \\ &= -36x^5 - 15x^2 \end{aligned}$$

$$e) f(t) = (6+t^{-2})(8t^{10} - 5t^3)$$

$$\begin{aligned} f'(t) &= (6+t^{-2})(80t^9 - 15t^2) + (8t^{10} - 5t^3)(-2t^{-3}) \\ &= 480t^9 - 90t^2 + 80t^7 - 15t^0 - 16t^7 + 10t^6 \\ &= 480t^9 + 64t^7 - 90t^2 - 5 \end{aligned}$$

$$f) f(t) = (at+b)(ct^2-d)$$

$$\begin{aligned} f'(t) &= (at+b)(2ct) + (ct^2-d)(a) \\ &= 2act^2 + 2bct + act^2 - ad \\ &= 3act^2 + 2bct - ad \end{aligned}$$

$$g) g(u) = \sqrt{u}(2 - u^2 + 5u^4)$$

$$\begin{aligned} g'(u) &= u^{\frac{1}{2}}(-2u + 20u^3) + (2 - u^2 + 5u^4)\left(\frac{1}{2}u^{-\frac{1}{2}}\right) \\ &= -2u^{\frac{3}{2}} + 20u^{\frac{7}{2}} + u^{\frac{1}{2}} - \frac{1}{2}u^{\frac{3}{2}} + \frac{5}{2}u^{\frac{7}{2}} \\ &= \frac{-5}{2}u^{\frac{3}{2}} + \frac{45}{2}u^{\frac{7}{2}} + u^{\frac{1}{2}} \end{aligned}$$

6-3 cont.

2. cont.

$$h) g(v) = (v - \sqrt{v})(v^2 + \sqrt{v})$$

$$g'(v) = (v - v^{\frac{1}{2}})(2v + \frac{1}{2}v^{-\frac{1}{2}}) + (v^2 + v^{\frac{1}{2}})(1 - \frac{1}{2}v^{-\frac{1}{2}})$$

$$= 2v^2 + \frac{1}{2}v^{\frac{1}{2}} - 2v^{\frac{3}{2}} - \frac{1}{2}v^{\frac{1}{2}} + v^2 - \frac{1}{2}v^{\frac{3}{2}} + v^{\frac{1}{2}} - \frac{1}{2}v^{\frac{1}{2}}$$

$$= 3v^2 - \frac{5}{2}v^{\frac{3}{2}} + \frac{3}{2}v^{\frac{1}{2}} - 1$$

3. a)  $y = (1-2x)(3x-4), x=2$  at  $x=2 \rightarrow -12(2)+11$   
 $y' = (1-2x)(3) + (3x-4)(-2)$   $-12+11$   
 $= 3-6x-6x+8$   $-13$   
 $= -13x+11$

b)  $y = (1-x+x^2)(x-2), x=1$  at  $x=1$   
 $y' = (1-x+x^2)(1) + (x-2)(-1+2x)$   $3(1)^2 - 6(1) + 3$   
 $= 1-x+x^2 - x + 2x^2 + 2 - 4x$   $3 - 6 + 3$   
 $= 3x^2 - 6x + 3$   $0$

c)  $y = x^4(4x^3+2), x=-1$  at  $x=-1$   
 $y' = x^4(12x^2) + (4x^3+2)(4x^3)$   $28(-1)^6 + 8(-1)^3$   
 $= 12x^6 + 16x^6 + 8x^3$   $28 - 8$   
 $= 28x^6 + 8x^3$   $20$

d)  $y = (1+x-2x^2)(3x^3+x-1), x=1$   
 $y' = (1+x-2x^2)(9x^2+1) + (3x^3+x-1)(1-4x)$   
 $= 9x^2+1+9x^3+x-18x^4-2x^2+3x^3-12x^4+x-4x^2-1+4x$   
 $= -30x^4+12x^3+3x^2+6x$   
at  $x=1 \rightarrow -30(1)^4+12(1)^3+3(1)^2+6(1)$   
 $-30+12+3+6$   
 $-9$

6-3 cont.

3. e)  $y = x^{-5}(1+x^{-1}), x=1$

$$y' = x^{-5}(-1x^{-2}) + (1+x^{-1})(-5x^{-6})$$
$$= -x^{-7} - 5x^{-6} - 5x^{-7}$$
$$= -6x^{-7} - 5x^{-6}$$

when  $x=1 \rightarrow -6(1)^{-7} - 5(1)^{-6}$

$$\frac{-6}{1^7} - \frac{5}{1^6}$$

$$-6 - 5$$

$$-11$$

f)  $y = (2-3\sqrt{x})(4-\sqrt{x}), x=4$

$$y' = (2-3x^{\frac{1}{2}})(-\frac{1}{2}x^{-\frac{1}{2}})' + (4-x^{\frac{1}{2}})(-3 \cdot \frac{1}{2}x^{-\frac{1}{2}})$$

$$= -x^{\frac{1}{2}} + \frac{3}{2}x^0 - \frac{6}{2}x^{\frac{1}{2}} + \frac{3}{2}x^0$$

$$= -7x^{\frac{1}{2}} + \frac{3}{2}$$

when  $x=4 \rightarrow \frac{-7}{\sqrt{4}} + 3$

$$= \frac{-7}{\sqrt{4}} + 3$$

$$\frac{-7}{2} + 3 = \frac{-1}{2}$$

4. a)  $f(x) = (6x^4 - 3x^2 + 1)(2 - x^3) \quad f'(1)$

$$f'(x) = (6x^4 - 3x^2 + 1)(-3x^2) + (2 - x^3)(24x^3 - 6x)$$
$$= -18x^6 + 9x^4 - 3x^2 + 48x^3 - 12x - 24x^6 + 6x^4$$
$$= -42x^6 + 15x^4 + 48x^3 - 3x^2 - 12x$$

$$f'(1) = -42(1)^6 + 15(1)^4 + 48(1)^3 - 3(1)^2 - 12(1)$$
$$= -42 + 15 + 48 - 3 - 12$$
$$= 6$$

b)  $f(x) = (6x^4 - 3x^2 + 1)(2 - x^3)$

$$= 12x^4 - 6x^7 - 6x^2 + 3x^5 + 2 - x^3$$

$$f'(x) = 48x^3 - 42x^6 - 12x + 15x^4 - 3x^2$$

$$f'(1) = 48(1)^3 - 42(1)^6 - 12(1) + 15(1)^4 - 3(1)^2$$
$$= 48 - 42 - 12 + 15 - 3 = 6$$