Calculus 6-4

1. a) $f(x) = \dfrac{x-1}{x+1}$

$$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$= \frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}$$

$$= \frac{2}{(x+1)^2}$$

b) $f(x) = \dfrac{2x-1}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(2) - (2x-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$$

$$= \frac{-2x^2+2x+2}{(x^2+1)^2}$$

c) $g(x) = \dfrac{x}{x^2+2x-1}$

$$g'(x) = \frac{(x^2+2x-1)(1) - x(2x+2)}{(x^2+2x-1)^2}$$

$$= \frac{x^2+2x-1-2x^2-2x}{(x^2+2x-1)^2}$$

$$= \frac{-x^2-1}{(x^2+2x-1)^2}$$

6-4 cont.

1. cont.

d) $g(x) = \dfrac{x^3-1}{x^2+x+1}$

$$g'(x) = \frac{(x^2+x+1)(3x^2) - (x^3-1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{3x^4+3x^3+3x^2-(2x^4+x^3-2x-1)}{(x^2+x+1)^2}$$

$$= \frac{3x^4+3x^3+3x^2-2x^4-x^3+2x+1}{(x^2+x+1)^2}$$

$$= \frac{x^4+2x^3+3x^2+2x+1}{(x^2+x+1)^2}$$

e) $y = \dfrac{\sqrt{x}}{x^2+1}$

$$y' = \frac{(x^2+1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(2x)}{(x^2+1)^2}$$

$$= \frac{\frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{\frac{3}{2}}}{(x^2+1)^2}$$

$$= \frac{-\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}}{(x^2+1)^2}$$

$$= \left(\frac{-3}{2}x^{\frac{3}{2}} + \frac{1}{2\sqrt{x}}\right) \cdot \frac{1}{(x^2+1)^2}$$

$$= \left(\frac{-3x^{\frac{3}{2}}x^{\frac{1}{2}}}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right)\frac{1}{(x^2+1)^2}$$

$$= \frac{-3x^2+1}{2\sqrt{x}} \cdot \frac{1}{(x^2+1)^2}$$

$$= \frac{-3x^2+1}{2\sqrt{x}(x^2+1)^2}$$

6-4 cont.

1. f) $y = \dfrac{\sqrt{x}+2}{\sqrt{x}-2}$

$y' = \dfrac{(x^{\frac{1}{2}}-2)(\frac{1}{2}x^{-\frac{1}{2}}) - (x^{\frac{1}{2}}+2)(\frac{1}{2}x^{-\frac{1}{2}})}{(\sqrt{x}-2)^2}$

$= \dfrac{\cancel{\frac{1}{2}x^0} - x^{-\frac{1}{2}} - \cancel{\frac{1}{2}x^0} - x^{-\frac{1}{2}}}{(\sqrt{x}-2)^2}$

$= \dfrac{-2x^{-\frac{1}{2}}}{(\sqrt{x}-2)^2}$

$= \dfrac{-2}{\sqrt{x}} \cdot \dfrac{1}{(\sqrt{x}-2)^2}$

$= \dfrac{-2}{\sqrt{x}(\sqrt{x}-2)^2}$

g) $f(t) = \dfrac{2t+1}{t^2-3t+4}$

$f'(t) = \dfrac{(t^2-3t+4)(2) - (2t+1)(2t-3)}{(t^2-3t+4)^2}$

$= \dfrac{2t^2-6t+8-(4t^2-6t+2t-3)}{(t^2-3t+4)^2}$

$= \dfrac{2t^2-6t+8-4t^2+4t+3}{(t^2-3t+4)^2}$

$= \dfrac{-2t^2-2t+11}{(t^2-3t+4)^2}$

h) $g(t) = \dfrac{2t^2+3t+1}{t-1}$

$g'(t) = \dfrac{(t-1)(4t+3) - (2t^2+3t+1)(1)}{(t-1)^2}$

$= \dfrac{4t^2+3t-4t-3-2t^2-3t-1}{(t-1)^2}$

$= \dfrac{2t^2-4t-4}{(t-1)^2}$

6-4 cont.

1. i) $f(x) = \dfrac{1}{x^4 - x^2 + 1}$

$f'(x) = \dfrac{(x^4 - x^2 + 1)(0) - (1)(4x^3 - 2x)}{(x^4 - x^2 + 1)^2}$

$= \dfrac{-4x^3 + 2x}{(x^4 - x^4 + 1)^2}$

j) $f(x) = \dfrac{ax + b}{cx + d}$

$f'(x) = \dfrac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2}$

$= \dfrac{\cancel{acx} + ad - \cancel{acx} - bc}{(cx + d)^2}$

$= \dfrac{ad - bc}{(cx + d)^2}$

k) $f(x) = \dfrac{x^6}{x^5 - 10}$

$f'(x) = \dfrac{(x^5 - 10)(6x^5) - x^6(5x^4)}{(x^5 - 10)^2}$

$= \dfrac{6x^{10} - 60x^5 - 5x^{10}}{(x^5 - 10)^2}$

$= \dfrac{x^{10} - 60x^5}{(x^5 - 10)^2}$

l) $f(x) = \dfrac{1 - x^{-1}}{x + 1}$

$f'(x) = \dfrac{(x + 1)(x^{-2}) - (1 - x^{-1})(1)}{(x + 1)^2}$

$= \dfrac{x^{-1} + x^{-2} - 1 + x^{-1}}{(x + 1)^2}$

$= \dfrac{2x^{-1} + x^{-2} - 1}{(x + 1)^2}$

$\dfrac{\frac{2}{x} + \frac{1}{x^2} - 1}{(x + 1)^2} \cdot \dfrac{x^2}{x^2}$

$\dfrac{2x + 1 - x^2}{x^2(x + 1)^2}$

6-4 cont.

2. a) $f(x) = \dfrac{2+x}{1-2x}$ domain: $1-2x \neq 0$

$1 \neq 2x$

$f'(x) = \dfrac{(1-2x)(1)-(2+x)(-2)}{(1-2x)^2}$ $\frac{1}{2} \neq x$ $\{x \mid x \neq \frac{1}{2}\}$

$= \dfrac{1-2x+4+2x}{(1-2x)^2}$

$= \dfrac{5}{(1-2x)^2}$

b) $f(x) = \dfrac{x}{x^2-1}$ domain: $x^2-1 \neq 0$

$x^2 \neq 1$

$f'(x) = \dfrac{(x^2-1)(1)-x(2x)}{(x^2-1)^2}$ $x \neq \pm 1$

$= \dfrac{x^2-1-2x^2}{(x^2-1)^2}$ $\{x \mid x \neq \pm 1\}$

$= \dfrac{-x^2-1}{(x^2-1)^2}$

c) $f(x) = \dfrac{1}{(x+1)(2x-3)}$

$f'(x) = \dfrac{\cancel{(x+1)(2x-3)\cdot 0} - 1\left[(x+1)(2)+(2x-3)(1)\right]}{(x+1)^2(2x-3)^2}$

$= \dfrac{-1(2x+2+2x-3)}{(x+1)^2(2x-3)^2}$ domain:

$= \dfrac{-2x-2-2x+3}{(x+1)^2(2x-3)^2}$ $x+1 \neq 0$ $2x-3 \neq 0$

$x \neq -1$ $2x \neq 3$

$= \dfrac{-4x+1}{(x+1)^2(2x-3)^2}$ $x \neq \dfrac{3}{2}$

$\{x \mid x \neq -1, \frac{3}{2}\}$

6-4 cont.

2. d) $f(x) = \dfrac{2x+1}{x^2+2x-3}$

$$f'(x) = \frac{(x^2+2x-3)(2) - (2x+1)(2x+2)}{(x^2+2x-3)^2}$$

$$= \frac{2x^2+4x-6-(4x^2+4x+2x+2)}{(x^2+2x-3)^2}$$

$$= \frac{2x^2+4x-6-4x^2-6x-2}{(x^2+2x-3)^2}$$

$$= \frac{-2x^2-2x-8}{(x^2+2x-3)^2}$$

domain:

$x^2+2x-3 \neq 0$

$(x+3)(x-1) \neq 0$

$x+3 \neq 0 \quad x-1 \neq 0$

$\{x \mid x \neq -3, 1\} \leftarrow x \neq -3 \quad x \neq 1$

e) $f(x) = \dfrac{x^2+2x}{x^4-1}$

$$f'(x) = \frac{(x^4-1)(2x+2) - (x^2+2x)4x^3}{(x^4-1)^2}$$

$$= \frac{2x^5+2x^4-2x-2-4x^5-8x^4}{(x^4-1)^2}$$

$$= \frac{-2x^5-6x^4-2x-2}{(x^4-1)^2}$$

domain:

$x^4-1 \neq 0$

$x^4 \neq 1$

$x \neq \pm 1$

$\{x \mid x \neq \pm 1\}$

f) $f(x) = \dfrac{x^2}{\sqrt{x}-3}$

$$f'(x) = \frac{(x^{\frac{1}{2}}-3)(2x) - x^2(\frac{1}{2}x^{-\frac{1}{2}})}{(\sqrt{x}-3)^2}$$

$$= \frac{2x^{\frac{3}{2}}-6x-\frac{1}{2}x^{\frac{3}{2}}}{(\sqrt{x}-3)^2}$$

$$= \frac{\frac{3}{2}x^{\frac{3}{2}}-6x}{(\sqrt{x}-3)^2} \cdot \frac{2}{2}$$

$$= \frac{3x^{\frac{3}{2}}-12x}{2(\sqrt{x}-3)^2}$$

domain:

$\sqrt{x}-3 \neq 0$

$\sqrt{x} \neq 3$

$x \neq 9$

$\{x \mid x \neq 9\}$

6-4 cont.

3. a) $y = \frac{x}{x-2}$, $(4,2)$

at $x=4$: $\frac{-2}{(4-2)^2} = \frac{-2}{2^2} = \frac{-2}{4} = \frac{-1}{2}$

$y' = \frac{(x-2)(1) - x(1)}{(x-2)^2}$

$= \frac{\cancel{x}-2-\cancel{x}}{(x-2)^2}$

$= \frac{-2}{(x-2)^2}$

$y-2 = -\frac{1}{2}(x-4)$

$2(y-2) = 2(-\frac{1}{2})(x-4)$

$2y-4 = -(x-4)$

$2y-4 = -x+4$

$x+2y-8=0$

b) $y = \frac{1+3x}{2-3x}$, $(1,-4)$

at $x=1$: $\frac{9}{(2-3\cdot1)^2} = \frac{9}{(-1)^2} = \frac{9}{1} = 9$

$y' = \frac{(2-3x)(3)-(1+3x)(-3)}{(2-3x)^2}$

$= \frac{6-\cancel{9x}+3+\cancel{9x}}{(2-3x)^2}$

$= \frac{9}{(2-3x)^2}$

$y+4 = 9(x-1)$

$y+4 = 9x-9$

$0 = 9x-y-13$

c) $y = \frac{1}{x^2+1}$, $(-2, \frac{1}{5})$

at $x=-2$: $\frac{-2(-2)}{[(-2)^2+1]^2} = \frac{4}{5^2} = \frac{4}{25}$

$y' = \frac{(x^2+1)(0)-(1)(2x)}{(x^2+1)^2}$

$= \frac{-2x}{(x^2+1)^2}$

$y-\frac{1}{5} = \frac{4}{25}(x+2)$

$25y-5 = 4(x+2)$

$25y-5 = 4x+8$

$0 = 4x-25y+13$

d) $y = \frac{x^3-1}{1+2x^2}$, $(1,0)$

at $x=1$: $\frac{2\cdot1^4+3\cdot1^2+4\cdot1}{(1+2\cdot1^2)^2} = \frac{2+3+4}{3^2} = \frac{9}{9} = 1$

$y' = \frac{(1+2x^2)(3x^2)-(x^3-1)(4x)}{(1+2x^2)^2}$

$= \frac{3x^2+6x^4-4x^4+4x}{(1+2x^2)^2}$

$= \frac{2x^4+3x^2+4x}{(1+2x^2)^2}$

$y-0 = 1(x-1)$

$y = x-1$