Calculus 6-5

1. a) $F(x) = (5-3x)^7$

$F'(x) = 7(5-3x)^6(-3)$

$= -21(5-3x)^6$

b) $F(x) = (2x^2+1)^{20}$

$F'(x) = 20(2x^2+1)^{19}(4x)$

$= 80x(2x^2+1)^{19}$

c) $G(x) = (x^3+x^2-2)^{\frac{3}{4}}$

$G'(x) = \frac{3}{4}(x^3+x^2-2)^{-\frac{1}{4}}(3x^2+2x)$

$= \frac{3(3x^2+2x)}{4(x^3+x^2-2)^{\frac{1}{4}}}$

$= \frac{9x^2+6x}{4\sqrt[4]{x^3+x^2-2}}$

d) $G(x) = \sqrt{x^4-x+1}$

$= (x^4-x+1)^{\frac{1}{2}}$

$G'(x) = \frac{1}{2}(x^4-x+1)^{-\frac{1}{2}}(4x^3-1)$

$= \frac{4x^3-1}{2\sqrt{x^4-x+1}}$

e) $y = \sqrt[4]{x^2+x}$

$= (x^2+x)^{\frac{1}{4}}$

$y' = \frac{1}{4}(x^2+x)^{-\frac{3}{4}}(2x+1)$

$= \frac{2x+1}{4(x^2+x)^{\frac{3}{4}}}$

f) $y = (1+3x+4x^2)^{-3}$

$y' = -3(1+3x+4x^2)^{-4}(3+8x)$

$= \frac{-3(3+8x)}{(1+3x+4x^2)^4}$

$= \frac{-9-24x}{(1+3x+4x^2)^4}$

6-5 cont.

1. g) $y = \dfrac{1}{(x^3+2x+1)^2}$

$= (x^3+2x+1)^{-2}$

$y' = -2(x^3+2x+1)^{-3}(3x^2+2)$

$= \dfrac{-2(3x^2+2)}{(x^3+2x+1)^3}$

$= \dfrac{-6x^2-4}{(x^3+2x+1)^3}$

h) $y = \dfrac{4}{\sqrt{9-x^2}}$

$= 4(9-x^2)^{-\frac{1}{2}}$

$y' = {}^{2}\not{4}\left(\dfrac{-1}{\not{2}}\right)(9-x^2)^{-\frac{3}{2}}(-2x)$

$= \dfrac{4x}{(9-x^2)^{\frac{3}{2}}}$

i) $y = (1+2\sqrt{x})^6$

$y' = 6(1+2\sqrt{x})^5 \not{2} \cdot \dfrac{1}{\not{2}} x^{-\frac{1}{2}}$

$= \dfrac{6(1+2\sqrt{x})^5}{\sqrt{x}}$

j) $y = \sqrt{x+\sqrt{x}}$

$y = (x+(x)^{\frac{1}{2}})^{\frac{1}{2}}$

$y' = \dfrac{1}{2}(x+\sqrt{x})^{-\frac{1}{2}} \cdot \left(1+\dfrac{1}{2}x^{-\frac{1}{2}}\right)$

$= \dfrac{1}{2\sqrt{x+\sqrt{x}}} \cdot \left(\dfrac{2\sqrt{x}}{2\sqrt{x}} + \dfrac{1}{2\sqrt{x}}\right)$

$= \dfrac{1}{2\sqrt{x+\sqrt{x}}} \cdot \dfrac{2\sqrt{x}+1}{2\sqrt{x}}$

$= \dfrac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$

k) $y = x - \sqrt[5]{1+x^5-6x^{10}}$

$= x - (1+x^5-6x^{10})^{\frac{1}{5}}$

$y' = 1 - \dfrac{1}{5}(1+x^5-6x^{10})^{-\frac{4}{5}}(5x^4-60x^9)$

$= 1 - \dfrac{(\not{5}x^4 - \not{60}^{12}x^9)}{\not{5}(1+x^5-6x^{10})^{\frac{4}{5}}}$

$= 1 - \dfrac{(x^4-12x^9)}{(1+x^5-6x^{10})^{\frac{4}{5}}}$

l) $y = x^2 + (x^2-1)^5$

$y' = 2x + 5(x^2-1)^4 2x$

$= 2x + 10x(x^2-1)^4$

6-5 cont.

2\. a) $F(x) = x\sqrt{x^2+1}$

$= x(x^2+1)^{\frac{1}{2}}$

$F'(x) = x\frac{1}{2}(x^2+1)^{-\frac{1}{2}}2x + \sqrt{x^2+1}\cdot 1$

$= \frac{x^2}{\sqrt{x^2+1}} + \sqrt{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$

$= \frac{x^2+x^2+1}{\sqrt{x^2+1}}$

$= \frac{2x^2+1}{\sqrt{x^2+1}}$

b) $F(x) = (2x+1)(4x-1)^5$

$F'(x) = (2x+1)5(4x-1)^4(4) + (4x-1)^5(2)$

$= 20(2x+1)(4x-1)^4 + 2(4x-1)^5$

$= (4x-1)^4[20(2x+1) + 2(4x-1)]$

$= (4x-1)^4(40x+20+8x-2)$

$= (4x-1)^4(48x+18)$

$= 6(4x-1)^4(8x+3)$

c) $G(x) = (x^2-1)^4(2-3x)$

$G'(x) = (x^2-1)^4(-3) + (2-3x)4(x^2-1)^3(2x)$

$= -3(x^2-1)^4 + 8x(2-3x)(x^2-1)^3$

$= (x^2-1)^3[-3(x^2-1) + 8x(2-3x)]$

$= (x^2-1)^3(-3x^2+3+16x-24x^2)$

$= (x^2-1)^3(-27x^2+16x+3)$

d) $G(x) = (x^4-x+1)^2(x^2-2)^3$

$G'(x) = (x^4-x+1)^2 3(x^2-2)^2 2x + (x^2-2)^3 2(x^4-x+1)(4x^3-1)$

$= 6x(x^4-x+1)^2(x^2-2)^2 + 2(x^2-2)^3(x^4-x+1)(4x^3-1)$

$= 2(x^4-x+1)(x^2-2)^2[3x(x^4-x+1) + (x^2-2)(4x^3-1)]$

$= 2(x^4-x+1)(x^2-2)^2(3x^5-3x^2+3x+4x^5-x^2-8x^3+2)$

$= 2(x^4-x+1)(x^2-2)^2(7x^5-8x^3-4x^2+3x+2)$

6-5 cont.

2. e) $F(x) = \dfrac{x}{\sqrt{2x+3}}$

$$F'(x) = \frac{\sqrt{2x+3}\cdot 1 - x\frac{1}{2}(2x+3)^{-\frac{1}{2}}(2)}{\sqrt{2x+3}^{\,2}}$$

$$= \frac{\sqrt{2x+3} - \dfrac{2x}{2\sqrt{2x+3}}}{2x+3}$$

$$= \frac{\sqrt{2x+3}\cdot\dfrac{\sqrt{2x+3}}{\sqrt{2x+3}} - \dfrac{x}{\sqrt{2x+3}}}{2x+3}$$

$$= \frac{(2x+3) - x}{\sqrt{2x+3}}\cdot\frac{1}{2x+3}$$

$$= \frac{x+3}{(2x+3)^{\frac{3}{2}}}$$

f) $f(t) = \dfrac{(1+2t)^5}{(3t^2-5)^2}$

$$f'(t) = \frac{(3t^2-5)^2\,5(1+2t)^4(2) - (1+2t)^5\,2(3t^2-5)(6t)}{((3t^2-5)^2)^2}$$

$$= \frac{10(3t^2-5)^2(1+2t)^4 - 12t(1+2t)^5(3t^2-5)}{(3t^2-5)^4}$$

$$= \frac{2(3t^2-5)(1+2t)^4\left[5(3t^2-5) - 6t(1+2t)\right]}{(3t^2-5)^4}$$

$$= \frac{2\cancel{(3t^2-5)}(1+2t)^4(15t^2-25-6t-12t^2)}{(3t^2-5)^{3}}$$

$$= \frac{2(1+2t)^4(3t^2-6t-25)}{(3t^2-5)^3}$$

6-5 cont.

2. g) $g(x) = \left(\frac{x+2}{x-2}\right)^3$

$g'(x) = 3\left(\frac{x+2}{x-2}\right)^2 \left[\frac{(x-2)\cdot 1 - (x+2)\cdot 1}{(x-2)^2}\right]$

$= 3\frac{(x+2)^2}{(x-2)^2} \cdot \frac{x-2-x-2}{(x-2)^2}$

$= \frac{3(x+2)^2(-4)}{(x-2)^4}$

$= \frac{-12(x+2)^2}{(x-2)^4}$

h) $h(t) = \left(\frac{t^2+1}{t+1}\right)^{10}$

$h'(t) = 10\left(\frac{t^2+1}{t+1}\right)^9 \left[\frac{(t+1)(2t) - (t^2+1)(1)}{(t+1)^2}\right]$

$= \frac{10(t^2+1)^9}{1\,(t+1)^9} \cdot \frac{2t^2+2t-t^2-1}{(t+1)^2}$

$= \frac{10(t^2+1)^9(t^2+2t-1)}{(t+1)^{11}}$

i) $y = \sqrt{\frac{x^2-1}{x^2+1}}$

$y' = \frac{1}{2}\left(\frac{x^2-1}{x^2+1}\right)^{-\frac{1}{2}} \left[\frac{(x^2+1)2x - (x^2-1)2x}{(x^2+1)^2}\right]$

$= \frac{1}{2} \frac{(x^2+1)^{\frac{1}{2}}}{(x^2-1)^{\frac{1}{2}}} \cdot \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$

$= \frac{(x^2+1)^{\frac{1}{2}}(4x)}{2(x^2-1)^{\frac{1}{2}}(x^2+1)^{2}}$ ($\frac{3}{2}$)

$= \frac{2x}{(x^2-1)^{\frac{1}{2}}(x^2+1)^{\frac{3}{2}}}$

$\left(\frac{x^2-1}{x^2+1}\right)^{-\frac{1}{2}} = \frac{1}{\frac{(x^2-1)^{\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}}} = 1 \cdot \frac{(x^2+1)^{\frac{1}{2}}}{(x^2-1)^{\frac{1}{2}}} = \frac{(x^2+1)^{\frac{1}{2}}}{(x^2-1)^{\frac{1}{2}}}$

6-5 cont.

2. j) $$y = \frac{(2x+3)^3}{\sqrt{4x-7}}$$

$$y' = \frac{(4x-7)^{\frac{1}{2}}\,3(2x+3)^2(2) - (2x+3)^3\,\frac{1}{2}(4x-7)^{-\frac{1}{2}}(4)}{\left[(4x-7)^{\frac{1}{2}}\right]^2}$$

$$= \frac{6(4x-7)^{\frac{1}{2}}(2x+3)^2 - 2(2x+3)^3(4x-7)^{-\frac{1}{2}}}{4x-7}$$

$$= \frac{2(2x+3)^2(4x-7)^{-\frac{1}{2}}\left[3(4x-7)-(2x+3)\right]}{4x-7}$$

$$= \frac{2(2x+3)^2}{(4x-7)^{\frac{1}{2}}}\,\frac{(12x-21-2x-3)}{(4x-7)}$$

$$= \frac{2(2x+3)^2(10x-24)}{(4x-7)^{\frac{3}{2}}}$$

$$= \frac{4(2x+3)^2(5x-12)}{(4x-7)^{\frac{3}{2}}}$$

k) $$y = 3\sqrt{x}\,(2x+1)^5 + \sqrt{4x-3}$$

$$y' = 3x^{\frac{1}{2}}\,5(2x+1)^4(2) + (2x+1)^5\,\frac{1}{2}\cdot 3x^{-\frac{1}{2}} + \frac{1}{2}(4x-3)^{-\frac{1}{2}}(4)$$

$$= 30x^{\frac{1}{2}}(2x+1)^4 + \frac{3}{2}x^{-\frac{1}{2}}(2x+1)^5 + 2(4x-3)^{-\frac{1}{2}}$$

$$= 30x^{\frac{1}{2}}(2x+1)^4 + \frac{3(2x+1)^5}{2x^{\frac{1}{2}}} + \frac{2}{(4x-3)^{\frac{1}{2}}}$$

$$= \frac{30x^{\frac{1}{2}}(2x+1)^4(2x^{\frac{1}{2}})}{2x^{\frac{1}{2}}} + \frac{3(2x+1)^5}{2x^{\frac{1}{2}}} + \frac{2}{(4x-3)^{\frac{1}{2}}}$$

$$= \frac{60x(2x+1)^4 + 3(2x+1)^5}{2x^{\frac{1}{2}}} + \frac{2}{(4x-3)^{\frac{1}{2}}}$$

$$= \frac{3(2x+1)^4(20x+2x+1)}{2x^{\frac{1}{2}}} + \frac{2}{(4x-3)^{\frac{1}{2}}}$$

$$= \frac{3(2x+1)^4(22x+1)}{2x^{\frac{1}{2}}} + \frac{2}{(4x-3)^{\frac{1}{2}}}$$

$$= \frac{3(2x+1)^4(22x+1)}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

6-5 cont.

2. l) $y = \sqrt{1+\sqrt[3]{x}}$

$$y = (1+x^{\frac{1}{3}})^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(1+x^{\frac{1}{3}})^{-\frac{1}{2}} \cdot \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{1}{6\sqrt{1+\sqrt[3]{x}}\,\sqrt[3]{x^2}}$$

m) $y = (t+\sqrt[3]{t+t^2})^{20}$

$$y' = 20(t+\sqrt[3]{t+t^2})^{19}\left[1+\frac{1}{3}(t+t^2)^{-\frac{2}{3}}(1+2t)\right]$$

$$= 20(t+\sqrt[3]{t+t^2})^{19}\left[1+\frac{(1+2t)}{3(t+t^2)^{\frac{2}{3}}}\right]$$

n) $y = \sqrt{x+\sqrt{x+\sqrt{x}}}$

$$y = \left[x+(x+x^{\frac{1}{2}})^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$$y' = \frac{1}{2}\left[x+(x+x^{\frac{1}{2}})^{\frac{1}{2}}\right]^{-\frac{1}{2}}\left[1+\frac{1}{2}(x+x^{\frac{1}{2}})^{-\frac{1}{2}}\left(1+\frac{1}{2}x^{-\frac{1}{2}}\right)\right]$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(1+\frac{1}{2\sqrt{x}}\right)\right]$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}+\frac{1}{4\sqrt{x+\sqrt{x}}\,\sqrt{x}}\right]$$

If you get rid of the fraction in the numerator of their answer you will get the same answer as I have:

their answer: $y' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}\right)$

$$\frac{\frac{2\sqrt{x}}{2\sqrt{x}}+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}} =$$

$$\frac{2\sqrt{x}+1}{2\sqrt{x}}\cdot\frac{1}{2\sqrt{x+\sqrt{x}}} =$$

$$\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}} =$$

$$\frac{2\sqrt{x}}{2\cdot 2\sqrt{x}\sqrt{x+\sqrt{x}}}+\frac{1}{4\sqrt{x}\sqrt{x+\sqrt{x}}} =$$

$$\frac{1}{2\sqrt{x+\sqrt{x}}}+\frac{1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$$

$$\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}+\frac{1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}\right]$$