# Calculus 6-6

1. a) $x^2 - y^2 = 1$

$\frac{d}{dx}(x^2 - y^2) = \frac{d}{dx}1$

$\frac{d}{dx}(x^2) - \frac{d}{dx}(y^2)\frac{dy}{dx} = 0$

$2x - 2y\frac{dy}{dx} = 0$

$2x = 2y\frac{dy}{dx}$

$\frac{2x}{2y} = \frac{dy}{dx}$

$\frac{x}{y} = \frac{dy}{dx}$

b) $x^3 + y^3 = 6$

$\frac{d}{dx}x^3 + \frac{d}{dx}y^3 = \frac{d}{dx}6$

$3x^2 + 3y^2\frac{dy}{dx} = 0$

$3y^2\frac{dy}{dx} = -3x^2$

$\frac{dy}{dx} = \frac{-3x^2}{3y^2}$

$\frac{dy}{dx} = -\frac{x^2}{y^2}$

c) $xy = 4$

$\frac{d}{dx}xy = \frac{d}{dx}4$

$x \cdot 1\frac{dy}{dx} + y \cdot 1 = 0$

$x\frac{dy}{dx} + y = 0$

$x\frac{dy}{dx} = -y$

$\frac{dy}{dx} = -\frac{y}{x}$

d) $x^2 + xy + y^2 = 1$

$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}y^2 = \frac{d}{dx}1$

$2x + x \cdot 1\frac{dy}{dx} + y \cdot 1 + 2y\frac{dy}{dx} = 0$

$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx}(x + 2y) = -2x - y$

$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$

e) $x^3 + y^3 = 6xy$

$\frac{d}{dx}x^3 + \frac{d}{dx}y^3 = \frac{d}{dx}6xy$

$3x^2 + 3y^2\frac{dy}{dx} = 6x \cdot 1\frac{dy}{dx} + 6 \cdot y$

$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$

$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$

$\frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$

$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$

$\frac{dy}{dx} = \frac{3(2y - x^2)}{3(y^2 - 2x)}$

$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$

# 6-6 cont.

1. f) $2xy^2 - y^3 = x^2$

$\frac{d}{dx} 2xy^2 - \frac{d}{dx} y^3 = \frac{d}{dx} x^2$

$2x \cdot 2y\frac{dy}{dx} + 2 \cdot y^2 - 3y^2\frac{dy}{dx} = 2x$

$4xy\frac{dy}{dx} + 2y^2 - 3y^2\frac{dy}{dx} = 2x$

$4xy\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 2x - 2y^2$

$\frac{dy}{dx}(4xy - 3y^2) = 2x - 2y^2$

$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy - 3y^2}$

g) $\sqrt{x} + \sqrt{y} = 1$

$\frac{d}{dx} x^{\frac{1}{2}} + \frac{d}{dx} y^{\frac{1}{2}} = \frac{d}{dx} 1$

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$2\sqrt{y} \cdot \frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \cdot 2\sqrt{y}$

$\frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}}$

$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

h) $\frac{2x}{x+y} = y$

$\frac{d}{dx}\frac{2x}{x+y} = \frac{d}{dx} y$

$\frac{(x+y)(2) - 2x(1 + \frac{dy}{dx})}{(x+y)^2} = \frac{dy}{dx}$

$\frac{2x + 2y - 2x - 2x\frac{dy}{dx}}{(x+y)^2} = \frac{dy}{dx}$

$\frac{2y - 2x\frac{dy}{dx}}{(x+y)^2} = \frac{dy}{dx}$

$\frac{2y}{(x+y)^2} - \frac{2x\frac{dy}{dx}}{(x+y)^2} = \frac{dy}{dx}$

$\frac{2y}{(x+y)^2} = \frac{dy}{dx} + \frac{2x\frac{dy}{dx}}{(x+y)^2}$

$\frac{2y}{(x+y)^2} = \frac{dy}{dx}\left(1 + \frac{2x}{(x+y)^2}\right)$

$\frac{2y}{(x+y)^2} = \frac{dy}{dx}\left(\frac{(x+y)^2}{(x+y)^2} + \frac{2x}{(x+y)^2}\right)$

$\frac{2y}{(x+y)^2} = \frac{dy}{dx}\left(\frac{(x+y)^2 + 2x}{(x+y)^2}\right)$

$\frac{2y}{(x+y)^2} \cdot \frac{(x+y)^2}{(x+y)^2 + 2x} = \frac{dy}{dx}$

$\frac{2y}{(x+y)^2 + 2x} = \frac{dy}{dx}$

6-6 cont

2. a) $x^2+4y^2=5$ at $(1,-1)$

$\frac{d}{dx}x^2+\frac{d}{dx}4y^2=\frac{d}{dx}5$

$2x+8y\frac{dy}{dx}=0$

$8y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-2x}{8y}$

$\frac{dy}{dx}=\frac{-x}{4y}$

Slope $=\frac{-1}{4(-1)}=\frac{-1}{-4}=\frac{1}{4}$

b) $x^4+y^4=17$ at $(2,1)$

$\frac{d}{dx}x^4+\frac{d}{dx}y^4=\frac{d}{dx}17$

$4x^3+4y^3\frac{dy}{dx}=0$

$4y^3\frac{dy}{dx}=-4x^3$

$\frac{dy}{dx}=\frac{-4x^3}{4y^3}$

$\frac{dy}{dx}=\frac{-x^3}{y^3}$

Slope $=\frac{-(2)^3}{1^3}=\frac{-8}{1}=-8$

c) $x^2+x^3y^2-y^3=13$ at $(1,-2)$

$\frac{d}{dx}x^2+\frac{d}{dx}(x^3y^2)-\frac{d}{dx}y^3=\frac{d}{dx}13$

$2x+x^3\,2y\frac{dy}{dx}+3x^2y^2-3y^2\frac{dy}{dx}=0$

$x^3\,2y\frac{dy}{dx}-3y^2\frac{dy}{dx}=-2x-3x^2y^2$

$\frac{dy}{dx}(2x^3y-3y^2)=-2x-3x^2y^2$

$\frac{dy}{dx}=\frac{-2x-3x^2y^2}{2x^3y-3y^2}$

Slope $=\frac{-2(1)-3(1)^2(-2)^2}{2(1)^3(-2)-3(-2)^2}$

$=\frac{-2-12}{-4-12}$

$=\frac{-14}{-16}$

$=\frac{7}{8}$

6-6 cont.

2. a) $y^2 = 2xy - 3$

$\frac{d}{dx} y^2 = \frac{d}{dx} 2xy - \frac{d}{dx} 3$

$2y\frac{dy}{dx} = 2x\left(1\frac{dy}{dx}\right) + 2y - 0$

$2y\frac{dy}{dx} - 2x\frac{dy}{dx} = 2y$

$\frac{dy}{dx}(2y - 2x) = 2y$

$\frac{dy}{dx} = \frac{2y}{2y - 2x}$

at (2,3)

slope $= \frac{2(3)}{2(3) - 2(2)}$

$= \frac{6}{6-4}$

$= \frac{6}{2} = 3$

e) $\sqrt{x+y} + \sqrt{xy} = 4$

$\frac{d}{dx}(x+y)^{\frac{1}{2}} + \frac{d}{dx}(xy)^{\frac{1}{2}} = \frac{d}{dx} 4$

$\frac{1}{2}(x+y)^{-\frac{1}{2}}\left(1 + \frac{dy}{dx}\right) + \frac{1}{2}(xy)^{-\frac{1}{2}}\left(x\frac{dy}{dx} + 1\cdot y\right) = 0$

$\frac{1 + \frac{dy}{dx}}{2\sqrt{x+y}} + \frac{x\frac{dy}{dx} + y}{2\sqrt{x}} = 0$

$\frac{1}{2\sqrt{x+y}} + \frac{\frac{dy}{dx}}{2\sqrt{x+y}} + \frac{x\frac{dy}{dx}}{2\sqrt{x}} + \frac{y}{2\sqrt{x}} = 0$

$\frac{\frac{dy}{dx}}{2\sqrt{x+y}} + \frac{x\frac{dy}{dx}}{2\sqrt{x}} = -\frac{1}{2\sqrt{x+y}} - \frac{y}{2\sqrt{x}}$

$\frac{dy}{dx}\left(\frac{1}{2\sqrt{x+y}} + \frac{x}{2\sqrt{x}}\right) = -\frac{1}{2\sqrt{x+y}} - \frac{y}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{-\frac{1}{2\sqrt{x+y}} - \frac{y}{2\sqrt{x}}}{\frac{1}{2\sqrt{x+y}} + \frac{x}{2\sqrt{x}}}$

at (2,2)

slope $= \frac{-\frac{1}{2\sqrt{2+2}} - \frac{2}{2\sqrt{2}}}{\frac{1}{2\sqrt{2+2}} + \frac{2}{2\sqrt{2}}}$

$= \frac{-\frac{1}{4} - \frac{2}{2\sqrt{2}}}{\frac{1}{4} + \frac{2}{2\sqrt{2}}}$

$= \frac{-1\left(\frac{1}{4} + \frac{2}{2\sqrt{2}}\right)}{\frac{1}{4} + \frac{2}{2\sqrt{2}}}$

$= -1$

f) $\frac{1}{x} + \frac{1}{y} = 1$

$\frac{d}{dx} x^{-1} + \frac{d}{dx} y^{-1} = \frac{d}{dx} 1$

$-x^{-2} - y^{-2}\frac{dy}{dx} = 0$

$-x^{-2} = y^{-2}\frac{dy}{dx}$

$\frac{-x^{-2}}{y^{-2}} = \frac{dy}{dx}$

$\frac{-y^2}{x^2} = \frac{dy}{dx}$

at $\left(\frac{3}{2}, 3\right)$

slope $= \frac{-3^2}{\left(\frac{3}{2}\right)^2}$

$= \frac{-9}{\frac{9}{4}}$

$= -9\ \frac{4}{9}$

$= -4$

6-6 cont.

3. a) $2x^2 - y^2 = 1$

$\frac{d}{dx}2x^2 - \frac{d}{dx}y^2 = \frac{d}{dx}1$

$4x - 2y\frac{dy}{dx} = 0$

$4x = 2y\frac{dy}{dx}$

$\frac{4x}{2y} = \frac{dy}{dx}$

at $(-1,-1)$

slope $= \frac{4(-1)}{2(-1)} = \frac{-4}{-2} = 2$

$y+1 = 2(x+1)$

$y+1 = 2x+2$

$y = 2x+1$

b) $x^3 + y^3 = 9$

$\frac{d}{dx}x^3 + \frac{d}{dx}y^3 = \frac{d}{dx}9$

$3x^2 + 3y^2\frac{dy}{dx} = 0$

$3y^2\frac{dy}{dx} = -3x^2$

$\frac{dy}{dx} = \frac{-3x^2}{3y^2}$

$\frac{dy}{dx} = \frac{-x^2}{y^2}$

at $(2,1)$

slope $= \frac{-2^2}{1^2} = \frac{-4}{1} = -4$

$y-1 = -4(x-2)$

$y-1 = -4x+8$

$y = -4x+9$

c) $y^5 + x^2y^3 = 10$

$\frac{d}{dx}y^5 + \frac{d}{dx}x^2y^3 = \frac{d}{dx}10$

$5y^4\frac{dy}{dx} + x^2 3y^2\frac{dy}{dx} + 2xy^3 = 0$

$5y^4\frac{dy}{dx} + 3x^2y^2\frac{dy}{dx} = -2xy^3$

$\frac{dy}{dx}(5y^4 + 3x^2y^2) = -2xy^3$

$\frac{dy}{dx} = \frac{-2xy^3}{5y^4 + 3x^2y^2}$

at $(-3,1)$

slope $= \frac{-2(-3)1^3}{5(1)^4 + 3(-3)^2(1)^2}$

$= \frac{6}{5+27} = \frac{6}{32} = \frac{3}{16}$

$y-1 = \frac{3}{16}(x+3)$

$y-1 = \frac{3}{16}x + \frac{9}{16}$

$16y - 16 = 3x + 9$

$0 = 3x - 16y + 25$

6-6 cont.

3. d) $(x+y)^3 = x^3 + y^3$

$\frac{d}{dx}(x+y)^3 = \frac{d}{dx}x^3 + \frac{d}{dx}y^3$

$3(x+y)^2\left(1+\frac{dy}{dx}\right) = 3x^2 + 3y^2\frac{dy}{dx}$

$3(x+y)^2 + 3(x+y)^2\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}$

$3(x+y)^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 3x^2 - 3(x+y)^2$

$\frac{dy}{dx}\left[3(x+y)^2 - 3y^2\right] = 3x^2 - 3(x+y)^2$

$\frac{dy}{dx} = \frac{3x^2 - 3(x+y)^2}{3(x+y)^2 - 3y^2}$

at $(-1, 1)$

$\text{slope} = \frac{3(-1)^2 - 3(-1+1)^2}{3(-1+1)^2 - 3(1)^2}$

$= \frac{3-0}{0-3}$

$= \frac{3}{-3}$

$= -1$

$y - 1 = -1(x+1)$

$y - 1 = -x - 1$

$x + y = 0$