

# Calculus 6-7

$$1. a) f(x) = x^5 - 4x^2 + 1$$

$$f'(x) = 5x^4 - 4 \cdot 2x + 0 \\ = 5x^4 - 8x$$

$$f''(x) = 5 \cdot 4x^3 - 8 \\ = 20x^3 - 8$$

$$b) g(x) = 7x^4 + 12x^3 - 4x + 8$$

$$g'(x) = 7 \cdot 4x^3 + 12 \cdot 3x^2 - 4 \\ = 28x^3 + 36x^2$$

$$g''(x) = 28 \cdot 3x^2 + 36 \cdot 2x \\ = 84x^2 + 72x$$

$$c) f(t) = 2t - \frac{1}{t+1}$$

$$= 2t - (t+1)^{-1}$$

$$f'(t) = 2 - (-1)(t+1)^{-2}(1) \\ = 2 + (t+1)^{-2} \\ = 2 + \frac{1}{(t+1)^2}$$

$$f''(t) = 0 + (-2)(t+1)^{-3}(1) \\ = \frac{-2}{(t+1)^3}$$

6-7 cont.

$$1. d) g(t) = \frac{4}{\sqrt{t}} \\ = 4t^{-\frac{1}{2}}$$

$$g'(t) = 4\left(-\frac{1}{2}\right)t^{-\frac{3}{2}} \\ = \frac{-2}{t^{\frac{3}{2}}}$$

$$g''(t) = -2\left(-\frac{3}{2}\right)t^{-\frac{5}{2}} \\ = \frac{3}{t^{\frac{5}{2}}}$$

$$e) y = (2x+1)^8$$

$$y' = 8(2x+1)^7(2) \\ = 16(2x+1)^7$$

$$y'' = 16 \cdot 7(2x+1)^6(2) \\ = 224(2x+1)^6$$

$$f) y = t^3 + \frac{1}{t^3} \\ = t^3 + t^{-3}$$

$$y' = 3t^2 + (-3)t^{-4} \\ = \frac{3t^2 - 3}{t^4}$$

$$y'' = 3 \cdot 2t - 3(-4)t^{-5} \\ = \frac{6t + 12}{t^5}$$

6-7 cont.

1. g)  $y = \sqrt{x^2+1}$   
 $= (x^2+1)^{\frac{1}{2}}$

$$y' = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)$$
$$= \frac{2x}{2(x^2+1)^{\frac{1}{2}}}$$
$$= \frac{x}{\sqrt{x^2+1}} \quad \text{or } x(x^2+1)^{-\frac{1}{2}}$$

$$y'' = x\left(-\frac{1}{2}\right)(x^2+1)^{-\frac{3}{2}}(2x) + 1(x^2+1)^{-\frac{1}{2}}$$
$$= \frac{-2x^2}{2(x^2+1)^{\frac{3}{2}}} + \frac{1}{(x^2+1)^{\frac{1}{2}}}$$
$$= \frac{-x^2}{(x^2+1)^{\frac{3}{2}}} + \frac{x^2+1}{(x^2+1)^{\frac{3}{2}}}$$
$$= \frac{1}{(x^2+1)^{\frac{3}{2}}}$$

h)  $y = \frac{t}{t-1}$   
 $= t(t-1)^{-1}$

$$y' = t(-1)(t-1)^{-2}(1) + 1(t-1)^{-1}$$
$$= -t(t-1)^{-2} + (t-1)^{-1}$$
$$= \frac{-t}{(t-1)^2} + \frac{1}{(t-1)}$$
$$= \frac{-t}{(t-1)^2} + \frac{(t-1)}{(t-1)^2}$$
$$= \frac{-1}{(t-1)^2} \quad \text{or } -1(t-1)^{-2}$$

$$y'' = -1(-2)(t-1)^{-3}(1)$$
$$= 2(t-1)^{-3}$$
$$= \frac{2}{(t-1)^3}$$

6-7 cont.

2. a)  $f(x) = 1 - 12x + 4x^2 - x^3$

$$f'(x) = 0 - 12 + 4 \cdot 2x - 3x^2$$

$$= -12 + 8x - 3x^2$$

$$f''(x) = 0 + 8 - 3 \cdot 2x$$

$$= 8 - 6x$$

$$f'''(x) = -6$$

b)  $f(x) = \frac{1}{x^5}$

$$= x^{-5}$$

$$f'(x) = -5x^{-6}$$

$$f''(x) = -5(-6)x^{-7}$$

$$= 30x^{-7}$$

$$f'''(x) = 30(-7)x^{-8}$$

$$= -210x^{-8}$$

$$= \frac{-210}{x^8}$$

c)  $y = \frac{3}{(4-x)^2}$

$$= 3(4-x)^{-2}$$

$$y' = 3(-2)(4-x)^{-3}(-1)$$

$$= 6(4-x)^{-3}$$

$$y'' = 6(-3)(4-x)^{-4}(-1)$$

$$= 18(4-x)^{-4}$$

$$y''' = 18(-4)(4-x)^{-5}(-1)$$

$$= 72(4-x)^{-5}$$

$$= \frac{72}{(4-x)^5}$$

6-7 cont.

$$\begin{aligned} 2. d) \quad y &= \sqrt{1+2x} \\ &= (1+2x)^{\frac{1}{2}} \\ y' &= \frac{1}{2}(1+2x)^{-\frac{1}{2}}(2) \\ &= (1+2x)^{-\frac{1}{2}} \\ y'' &= -\frac{1}{2}(1+2x)^{-\frac{3}{2}}(2) \\ &= -(1+2x)^{-\frac{3}{2}} \\ y''' &= -(-\frac{3}{2})(1+2x)^{-\frac{5}{2}}(2) \\ &= 3(1+2x)^{-\frac{5}{2}} \end{aligned}$$

$$3. \quad y = x^5 + x^4 + x^3 + x^2 + x + 1$$

$$y' = 5x^4 + 4x^3 + 3x^2 + 2x + 1$$

$$y'' = 20x^3 + 12x^2 + 6x + 2$$

$$y''' = 60x^2 + 24x + 6$$

$$y^{(4)} = 120x + 24$$

$$y^{(5)} = 120$$

$$y^{(6)} = 0$$

$$\begin{aligned} f''(2) &= \frac{-9(2)^4}{4(1+2^3)^{\frac{3}{2}}} + \frac{3(2)}{(1+2^3)^{\frac{1}{2}}} \\ &= \frac{-144}{4\sqrt{9^3}} + \frac{6}{\sqrt{9}} \end{aligned}$$

$$4. \quad f(x) = \sqrt{1+x^3} \\ = (1+x^3)^{\frac{1}{2}}$$

$$\begin{aligned} f'(x) &= \frac{1}{2}(1+x^3)^{-\frac{1}{2}}(3x^2) \\ &= \frac{3}{2}x^2(1+x^3)^{-\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} f''(x) &= \frac{3}{2}\left(-\frac{1}{2}\right)x^2(1+x^3)^{-\frac{3}{2}}(3x^2) + \frac{3}{2}2x(1+x^3)^{-\frac{1}{2}} \\ &= \frac{-9x^4}{4(1+x^3)^{\frac{3}{2}}} + \frac{3x}{(1+x^3)^{\frac{1}{2}}} \end{aligned}$$

$$\begin{aligned} &= \frac{-144}{108} + \frac{6 \cdot 36}{3 \cdot 36} \\ &= \frac{-144}{108} + \frac{216}{108} = \frac{72}{108} = \frac{2}{3} \end{aligned}$$

6-7 cont.

$$5. g(x) = \frac{1}{\sqrt{3x+4}}$$
$$= (3x+4)^{-\frac{1}{2}}$$

$$g'(x) = -\frac{1}{2} (3x+4)^{-\frac{3}{2}} (3)$$
$$= -\frac{3}{2} (3x+4)^{-\frac{3}{2}}$$

$$g''(x) = -\frac{3}{2} \left(-\frac{3}{2}\right) (3x+4)^{-\frac{5}{2}} (3)$$
$$= \frac{27}{4} (3x+4)^{-\frac{5}{2}}$$

$$g'''(x) = \frac{27}{4} \left(-\frac{5}{2}\right) (3x+4)^{-\frac{7}{2}} (3)$$
$$= -\frac{405}{8} (3x+4)^{-\frac{7}{2}}$$
$$= -\frac{405}{8(3x+4)^{\frac{7}{2}}}$$

$$g'''(4) = \frac{-405}{8(3 \cdot 4 + 4)^{\frac{7}{2}}}$$
$$= \frac{-405}{8 \sqrt{16^7}}$$
$$= \frac{-405}{8 \cdot 4^7}$$
$$= \frac{-405}{8 \cdot 16384}$$
$$= \frac{-405}{131072}$$