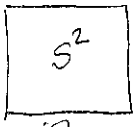
Calculus 7-5

1. square: $A = s^2$ $\quad \frac{ds}{dt} = 0.8\ m/min$ (rate of side growth) $\quad s = 3\ m$



$\frac{dA}{dt} = \frac{dA}{ds}\frac{ds}{dt}$ chain rule

$\frac{dA}{dt} = 2s\frac{ds}{dt}$ differentiate with respect to t (length of side)

$\frac{dA}{dt} = 2(3)(0.8)$ substitute a value for s and $\frac{ds}{dt}$

$\frac{dA}{dt} = 4.8\ m^2/min$

2. cube: $V = s^3$ $\quad \frac{dV}{dt} = 144\ cm^3/min$ (rate of volume growth) $\quad s = 4\ cm$

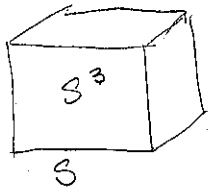
$\frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt}$

$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$

$144 = 3(4^2)\frac{ds}{dt}$

$144 = 48\frac{ds}{dt}$

$\frac{ds}{dt} = 3\ cm/s$



3. circle: $A = \pi r^2$ $\quad \frac{dr}{dt} = 25\ cm/s$ $\quad t = 4\ s$

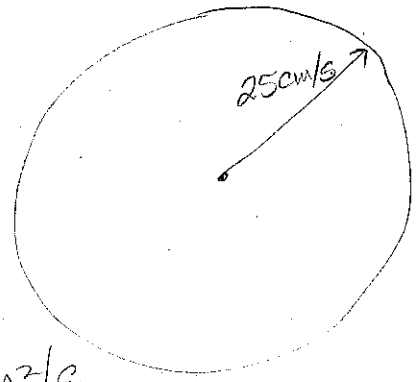
* r at 4s = 4·25 = 100cm

$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(100)(25)$

$\frac{dA}{dt} = 5000\pi$ or $15707.96\ cm^2/s$



7-5 cont.

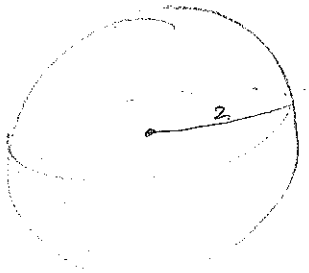
4. sphere: $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 8 m^3/min$ $\quad d = 2$, $2r = 2$, $r = 1$ { diameter = 2 radius

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$8 = 4\pi (1^2) \frac{dr}{dt}$$



$$\frac{8}{4\pi} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{2}{\pi} \text{ or } 0.6366 \text{ m/min}$$

5. sphere: $SA = 4\pi r^2$ $\quad \frac{dA}{dt} = -0.5 \text{ cm}^2/\text{min}$ $\quad r = 4$

$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$$

$$\frac{dA}{dt} = 4\pi 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$-0.5 = 8\pi (4) \frac{dr}{dt}$$

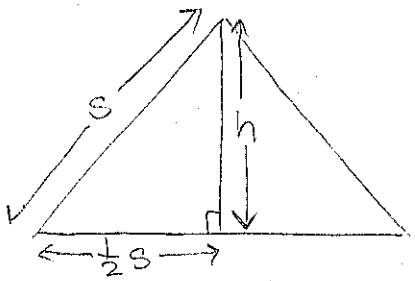
$$-0.5 = 32\pi \frac{dr}{dt}$$

$$\frac{-0.5}{32\pi} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = -0.0050 \text{ cm/min}$$

7-5 Cont.

6.



$s^2 = h^2 + \left(\frac{s}{2}\right)^2$

$s^2 = h^2 + \frac{s^2}{4}$

$s^2 - \frac{s^2}{4} = h^2$

$\frac{4s^2}{4} - \frac{s^2}{4} = h^2$

$\frac{3s^2}{4} = h^2$

$\frac{\sqrt{3}s}{4} = h$

$\frac{ds}{dt} = -2\ cm/s$

$A = 100\ cm^2$

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \cdot s \cdot \frac{\sqrt{3}s}{2}$

$A = \frac{s^2\sqrt{3}}{4}$

$\frac{dA}{ds} = \frac{2s\sqrt{3}}{4 \cdot 2}$

$\frac{dA}{ds} = \frac{s\sqrt{3}}{2}$

$100 = \frac{s^2\sqrt{3}}{4}$

$400 = s^2\sqrt{3}$

$\frac{400}{\sqrt{3}} = s^2$

$\frac{20}{\sqrt{3}} = s$

$\frac{20}{3^{\frac{1}{4}}} = s$

$\frac{dA}{dt} = \frac{dA}{ds} \cdot \frac{ds}{dt}$

$\frac{dA}{dt} = \frac{s\sqrt{3}}{2} \cdot (-2)$

$\frac{dA}{dt} = -s\sqrt{3}$

$\frac{dA}{dt} = -\frac{20}{3^{\frac{1}{4}}} \cdot 3^{\frac{1}{2}}$

$\frac{dA}{dt} = -20 \cdot \frac{3^{\frac{1}{2}}}{3^{\frac{1}{4}}}$

$\frac{dA}{dt} = -20 \cdot 3^{\frac{1}{4}}$