# 8.3 Increasing and Decreasing Functions

1. a) $f(x) = 12 + x - x^2$
$f'(x) = 1 - 2x$

$1 - 2x > 0$
$-2x > -1$
$\frac{-2x}{-2} < \frac{-1}{2}$
$x < \frac{1}{2}$ $\quad (-\infty, \frac{1}{2})$

b) $f(x) = x^4$
$f'(x) = 4x^3$

$4x^3 > 0$
$x^3 > 0$
$x > 0$ $\quad (0, \infty)$

c) $g(x) = x^3 - 3x + 2$
$g'(x) = 3x^2 - 3$

$3x^2 - 3 > 0$
$3x^2 > 3$
$x^2 > 1$
$x^2 - 1 > 0$

for $x^2 = 1$
$x = \pm 1$

| | $g'(x)$ |
|---|---|
| $x < -1$ | + |
| $-1 < x < 0$ | − |
| $0 < x < 1$ | − |
| $x > 1$ | + |

$(-\infty, -1), (1, \infty)$

d) $y = 2x^3 - 3x^2$
$y' = 6x^2 - 6x$

$6x^2 - 6x > 0$
$x^2 - x > 0$
$x(x-1) > 0$

$x = 0, 1$

| | $x$ | $x-1$ | $y'$ |
|---|---|---|---|
| $x < 0$ | − | − | + |
| $0 < x < 1$ | + | − | − |
| $x > 1$ | + | + | + |

$(-\infty, 0), (1, \infty)$

2. a) $f(x) = x^2 + x^3$
$f'(x) = 2x + 3x^2$

$2x + 3x^2 < 0$
$x(2 + 3x) < 0$

$x = 0, \quad 2 + 3x = 0$
$3x = -2$
$x = -\frac{2}{3}$

| | $x$ | $2+3x$ | $f'(x)$ |
|---|---|---|---|
| $x < -\frac{2}{3}$ | − | − | + |
| $-\frac{2}{3} < x < 0$ | − | + | − |
| $x > 0$ | + | + | + |

$(-\frac{2}{3}, 0)$

8.3 cont.

2. b) $g(x) = 2x^3 - 3x^2 - 36x + 62$

$g'(x) = 6x^2 - 6x - 36$

$6x^2 - 6x - 36 < 0$

$x^2 - x - 6 < 0$

$(x-3)(x+2) < 0$

$x \neq 3, -2$

| | $x-3$ | $x+2$ | $g(x)$ |
|---|---|---|---|
| $x < -2$ | $-$ | $-$ | $+$ |
| $-2 < x < 3$ | $-$ | $+$ | $-$ |
| $x > 3$ | $+$ | $+$ | $+$ |

$(-2, 3)$

c) $h(x) = (1-x^2)^2$

$h'(x) = 2(1-x^2)(-2x)$

$h'(x) = -4x(1-x^2)$

$-4x(1-x^2) < 0$

$1 - x^2 = 0$

$1 = x^2$

$\pm 1 = x, \quad x = 0$

| | $-4x$ | $1-x^2$ | $h'(x)$ |
|---|---|---|---|
| $x < -1$ | $+$ | $-$ | $-$ |
| $-1 < x < 0$ | $+$ | $+$ | $+$ |
| $0 < x < 1$ | $-$ | $+$ | $-$ |
| $x > 1$ | $-$ | $-$ | $+$ |

$(-\infty, -1), (0, 1)$

d) $F(x) = 4x + x^4$

$F'(x) = 4 + 4x^3$

$4 + 4x^3 < 0$

$1 + x^3 < 0$

$1 + x^3 = 0$

$x^3 = -1$

$x = -1$

| | $1+x^3$ |
|---|---|
| $x < -1$ | $-$ |
| $x > -1$ | $+$ |

$(-\infty, -1)$

3. a) $f(x) = 3x^2 - 18x + 1$

$f'(x) = 6x - 18$

$6x - 18 = 0$

$6x = 18$

$x = 3$

| | $6x-18$ | |
|---|---|---|
| $x < 3$ | $-$ | decreasing $(-\infty, 3)$ |
| $x > 3$ | $+$ | increasing $(3, \infty)$ |

8.3 cont.

3) b) $f(x) = 2x^3 - 9x^2 - 60x + 82$

$f'(x) = 6x^2 - 18x - 60$

$6x^2 - 18x - 60 = 0$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x-5=0, \quad x+2=0$

$x = 5 \quad x = -2$

| | $x-5$ | $x+2$ | $f'(x)$ |
|---|---|---|---|
| $x<-2$ | − | − | + |
| $-2<x<5$ | − | + | − |
| $x>5$ | + | + | + |

increase $(-\infty, -2), (5, \infty)$

decrease $(-2, 5)$

c) $g(x) = x^3(x-1)^4$

$g'(x) = 3x^2(x-1)^4 + x^3 \cdot 4(x-1)^3 \cdot 1$

$g'(x) = 3x^2(x-1)^4 + 4x^3(x-1)^3$

$g'(x) = (x-1)^3[3x^2(x-1) + 4x^3]$

$g'(x) = (x-1)^3(3x^3 - 3x^2 + 4x^3)$

$g'(x) = (x-1)^3(7x^3 - 3x^2)$

$g'(x) = (x-1)^3 x^2(7x-3)$

$(x-1)^3 = 0, \quad x^2 = 0, \quad 7x-3 = 0$

$x-1=0 \quad x=0 \quad 7x=3$

$x = 1 \quad x = \frac{3}{7}$

increase $(-\infty, \frac{3}{7}), (1, \infty)$

decrease $(\frac{3}{7}, 1)$

| | $(x-1)^3$ | $x^2$ | $7x-3$ | $g'(x)$ |
|---|---|---|---|---|
| $x<0$ | − | + | − | + |
| $0<x<\frac{3}{7}$ | − | + | − | + |
| $\frac{3}{7}<x<1$ | − | + | + | − |
| $x>1$ | + | + | + | + |

d) $h(x) = \frac{x-1}{x+1}$

$h'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$

$h'(x) = \frac{x+1-x+1}{(x+1)^2}$

$h'(x) = \frac{2}{(x+1)^2}$

$x+1 \neq 0$

$x \neq -1$

| | $(x+1)^2$ | $h'(x)$ |
|---|---|---|
| $x<-1$ | + | + |
| $x>-1$ | + | + |

increasing $(-\infty, -1), (-1, \infty)$