8-4 Maximum and Minimum Values

1. i) a) $f(7)=5$
   b) $f(2)=-2$
   c) $f(0)=2$, $f(4)=3$
   d) $f(-2)=1$, $f(6)=1$

   ii) a) $f(4)=5$
   b) $f(-3)=-2$
   c) $f(1)=4$, $f(8)=4$
   d) $f(2)=2$, $f(6)=1$

2. See next page.

3. a) $f(x)=17-6x+12x^2$
$f'(x)=-6+24x$
$=-6(1-4x)$

$1-4x=0$
$1=4x$
$\frac{1}{4}=x$

b) $f(x)=x^3-3x+2$
$f'(x)=3x^2-3$
$=3(x^2-1)$
$=3(x-1)(x+1)$

$3(x-1)(x+1)=0$
$\therefore x-1=0$ or $x+1=0$
$x=1$ $x=-1$

c) $g(x)=x^4-4x^3-8x^2-1$
$g'(x)=4x^3-12x^2-16x$
$=4x(x^2-3x-4)$
$=4x(x-4)(x+1)$

$4x(x-4)(x+1)=0$
$\therefore 4x=0$ or $x-4=0$ or $x+1=0$
$x=0$ $x=4$ $x=-1$

d) $g(x)=3x^4-16x^3+6x^2+72x+8$
$g'(x)=12x^3-48x^2+12x+72$
$=12(x^3-4x^2+x+6)$

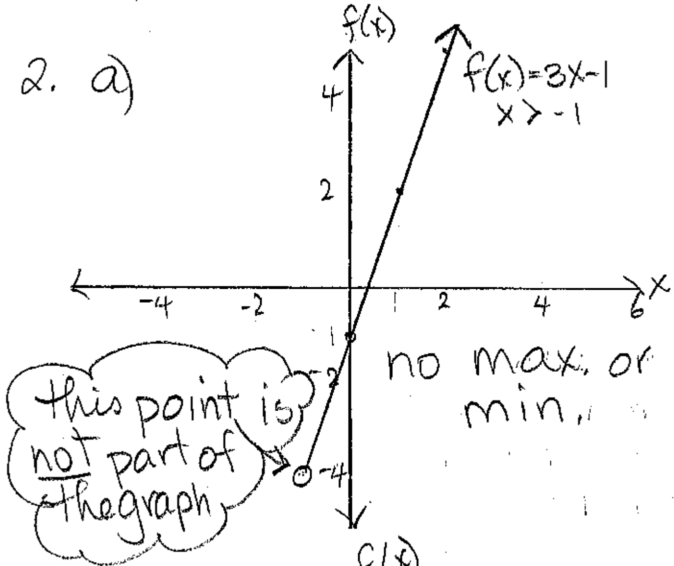
factors of 6: $\pm1, \pm2, \pm3, \pm6$
try: 2
$2^3-4\cdot2^2+2+6=$
$8-16+2+6=$
0

It's factorable: divide by $x-2$.

see 3rd page to continue

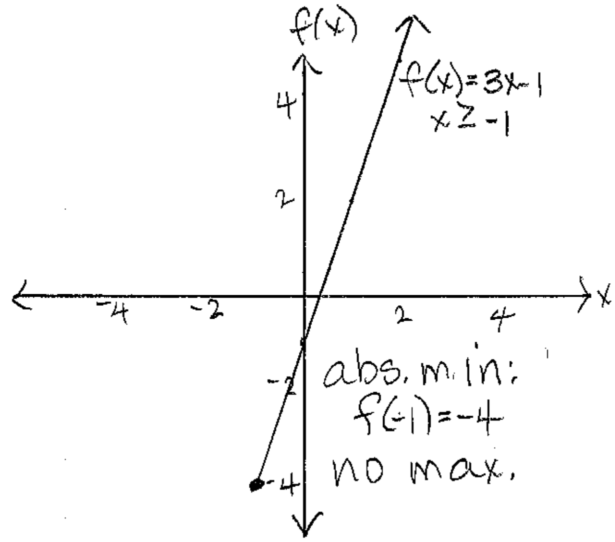
8-4 cont.

2. a)



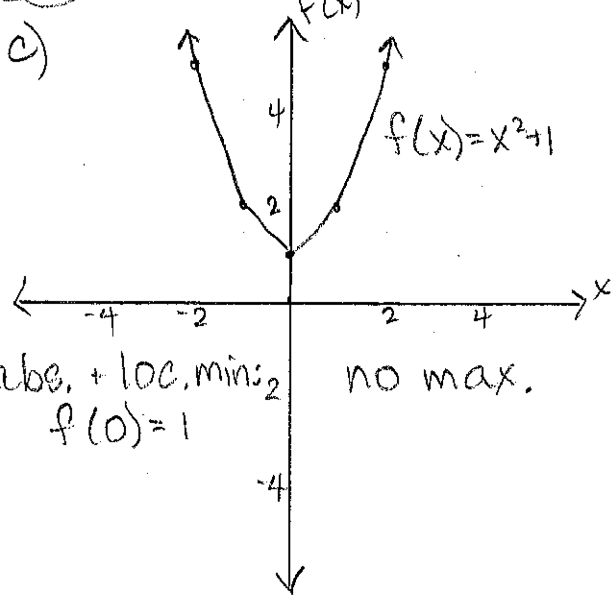
no max. or min.

b)



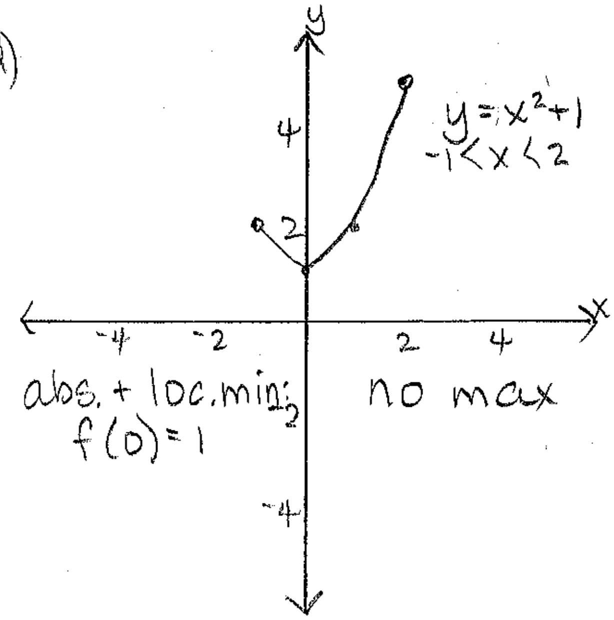
abs. min: $f(-1)=-4$
no max.

c)



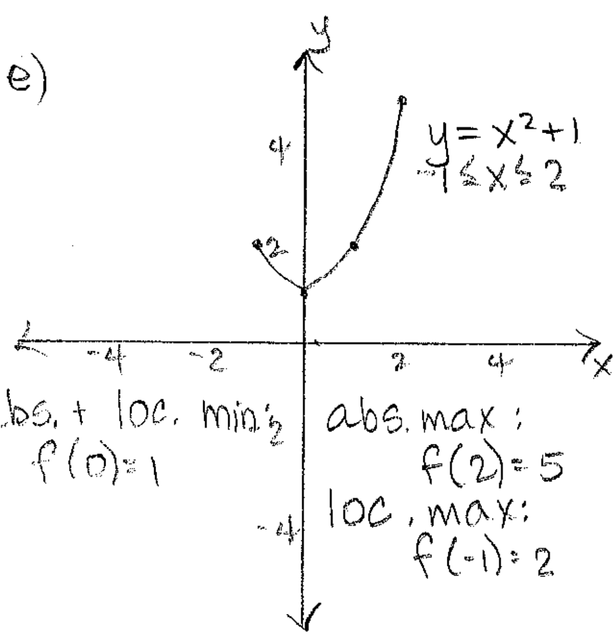
abs. + loc. min: $f(0)=1$
no max.

d)



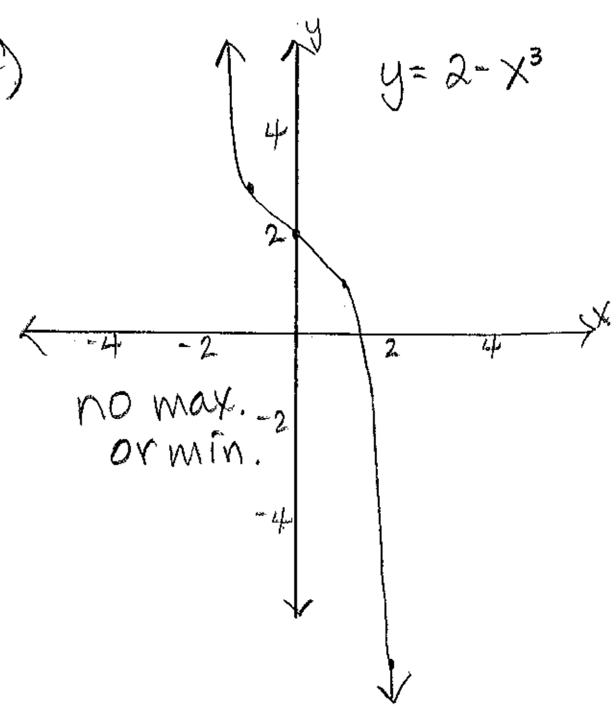
abs. + loc. min: $f(0)=1$
no max

e)



abs. + loc. min: $f(0)=1$
abs. max: $f(2)=5$
loc. max: $f(-1)=2$

f)



no max. or min.

8-4 cont.

3. d) cont.

$$\begin{array}{r} x^2-2x-3 \\ x-2\overline{)\,x^3-4x^2+x+6} \\ \underline{x^3-2x^2} \\ -2x^2+x \\ \underline{-2x^2+4x} \\ -3x+6 \\ \underline{-3x+6} \\ 0 \end{array}$$

factors:

$(x-2)(x^2-2x-3)$

$(x-2)(x-3)(x+1)$

$\therefore\ x-2=0$ or $x-3=0$ or $x+1=0$

$x=2 \qquad x=3 \qquad x=-1$

4. a) $f(x)=2x^2-8x+1$, $0\le x\le 3$ end points

$f'(x)=4x-8$

$=4(x-2)$

$4(x-2)=0$

$\therefore\ x-2=0$

$x=2$ critical number

$f(2)=2(2)^2-8(2)+1$

$=8-16+1$

$=-7$

$f(0)=2(0)^2-8(0)+1$

$=0-0+1$

$=1$

$f(3)=2(3)^2-8(3)+1$

$=18-24+1$

$=-5$

abs. max : $f(0)=1$

abs. min : $f(2)=-7$

b) $f(x)=3+2(x+1)^2$, $-3\le x\le 2$

$=3+2(x^2+2x+1)$

$=3+2x^2+4x+2$

$=2x^2+4x+5$

$f'(x)=4x+4$

$=4(x+1)$

$\therefore\ x+1=0$

$x=-1$

$f(-1)=3+2(-1+1)^2$

$=3+0$

$=3$

$f(-3)=3+2(-3+1)^2$

$=3+2(-2)^2$

$=3+8$

$=11$

see next page

8-4 Cont.

4. b) $f(2) = 3+2(2+1)^2$
$= 3+2(3)^2$
$= 3+18$
$= 21$

abs. max: $f(2) = 21$
abs. min: $f(-1) = 3$

c) $f(x) = 2x^3 - 3x^2, \quad -2 \le x \le 2$
$f'(x) = 6x^2 - 6x$
$= 6x(x-1)$
$6x(x-1) = 0$
$\therefore\ 6x = 0$ or $x-1 = 0$
$x = 0 \qquad x = 1$

$f(0) = 2\cdot 0^3 - 3\cdot 0^2$
$= 0-0$
$= 0$

$f(1) = 2\cdot 1^3 - 3\cdot 1^2$
$= 2-3$
$= -1$

$f(-2) = 2(-2)^3 - 3(-2)^2$
$= -16-12$
$= -28$

$f(2) = 2(2)^3 - 3(2)^2$
$= 16-12$
$= 4$

abs. max.: $f(2) = 4$ \qquad abs. min.: $f(-2) = -28$

d) $f(x) = 2x^3 - 3x^2 - 36x + 62, \quad -3 \le x \le 4$
$f'(x) = 6x^2 - 6x - 36$
$= 6(x^2 - 1 - 6)$
$= 6(x-3)(x+2)$
$(x-3)(x+2) = 0$
$\therefore\ x-3 = 0$ or $x+2 = 0$
$x = 3 \qquad x = -2$

$f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 62$
$= 54 - 27 - 108 + 62$
$= -19$

$f(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) + 62$
$= -16 - 12 + 72 + 62$
$= 106$

$f(-3) = 2(-3)^3 - 3(-3)^2 - 36(-3) + 62$
$= 54 - 27 + 108 + 62$
$= 89$

$f(4) = 2(4)^3 - 3(4)^2 - 36(4) + 62$
$= 128 - 48 - 144 + 62$
$= -2$

abs. max.: $f(-2) = 106$
abs. min.: $f(3) = -19$

8-4 cont.

4. e) $f(x) = x^4 - 2x^2 + 16, \quad -3 \le x \le 2$

$f'(x) = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x-1)(x+1)$

$4x(x-1)(x+1) = 0$

$\therefore\ 4x = 0$ or $x - 1 = 0$ or $x + 1 = 0$

$x = 0 \qquad x = 1 \qquad x = -1$

$f(2) = 2^4 - 2 \cdot 2^2 + 16$

$= 16 - 8 + 16$

$= 24$

$f(0) = 0^4 - 2 \cdot 0^2 + 16$

$= 0 - 0 + 16$

$= 16$

$f(1) = 1^4 - 2 \cdot 1^2 + 16$

$= 1 - 2 + 16$

$= -1 + 16$

$= 15$

$f(-1) = (-1)^4 - 2(-1)^2 + 16$

$= 1 - 2 + 16$

$= 15$

$f(-3) = (-3)^4 - 2(-3)^2 + 16$

$= 81 - 18 + 16$

$= 79$

abs. max.: $f(-3) = 79$

abs. min.: $f(-1) = 15$

$f(1) = 15$

f) $f(x) = x^5 + 3x^3 + x, \quad -1 \le x \le 2$

$f'(x) = 5x^4 + 9x^2 + 1$

can't be factored

$x^2 = \frac{-9 \pm \sqrt{81 - 4(5)(1)}}{10}$

$= \frac{-9 \pm \sqrt{61}}{10}$

$= -8.22$ or -9.78

which is impossible because it is x^2 so no critical points → graph it and this will be confirmed

$f(-1) = (-1)^5 + 3(-1)^3 + (-1)$

$= -1 - 3 - 1$

$= -5$

$f(2) = 2^5 + 3 \cdot 2^3 + 2$

$= 32 + 24 + 2$

$= 58$

abs. max.: $f(2) = 58$

abs. min.: $f(-1) = -5$