8-6 Applied Maximum and Minimum Problems

1. two numbers: $x > 0$ and $y > 0$

minimize $S = 2x + y$ $\qquad$ $xy = 200$

$S = 2x + \frac{200}{x}$ $\longleftarrow$ $y = \frac{200}{x}$

$= 2x + 200x^{-1}$

$S' = 2 + (-1)200x^{-2}$

$= 2 - \frac{200}{x^2}$

$= \frac{2x^2}{x^2} - \frac{200}{x^2}$

$= \frac{2x^2 - 200}{x^2}$

$\frac{2x^2 - 200}{x^2} = 0$

$2x^2 - 200 = 0$

$2x^2 = 200$

$x^2 = 100$

$x = \pm 10$

but $x > 0$ so $x = 10$

	$\frac{2x^2-200}{x^2}$	interval
$0 < x < 10$	$-$	decr.
$x > 10$	$+$	incr.

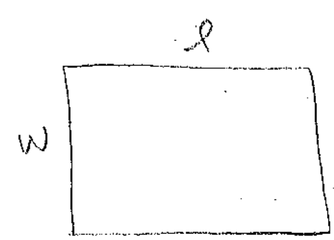
$x = 10$ is a min.

$y = \frac{200}{10}$

$y = 20$

The two numbers are 10 and 20.

2.



minimize $P = 2w + 2l$.

$A = lw$ (say area is 100)

$100 = lw$

$\frac{100}{w} = l$

$P = 2w + 2\left(\frac{100}{w}\right)$

$= 2w + \frac{200}{w}$

$= 2w + 200w^{-1}$

$P' = 2 + 200(-1)w^{-2}$

$= 2 - \frac{200}{w^2}$

$= \frac{2w^2 - 200}{w^2}$

see next page

8-6 cont.

2. cont.

$0 = \frac{2w^2 - 200}{w^2}$

$0 = 2w^2 - 200$

$200 = 2w^2$

$100 = w^2$

$\pm 10 = w$ but $w > 0$

$10 = w$

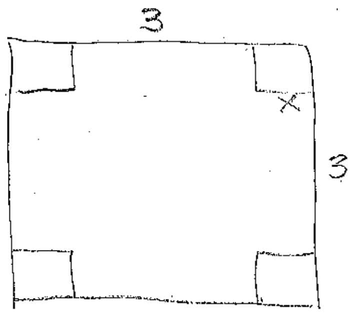
	$\frac{2w^2-200}{w^2}$	interval
$0 < w < 10$	$-$	decr.
$w > 10$	$+$	incr.

$w = 10$ is a min.

$100 = l\,10$

$10 = l$

So the perimeter is a minimum when $w = l = 10$.

3. 

maximize $V = (3-2x)(3-2x)x$

$= (9 - 6x - 6x + 4x^2)x$

$= (9 - 12x + 4x^2)x$

$= 9x - 12x^2 + 4x^3$

$V' = 9 - 24x + 12x^2$

$= 3(4x^2 - 8x + 3)$

$= 3(4x^2 - 2x - 6x + 3)$

$= 3[2x(2x-1) - 3(2x-1)]$

$= 3(2x-1)(2x-3)$

$3(2x-1)(2x-3) = 0$

$\therefore 2x - 1 = 0, \quad 2x - 3 = 0$

$2x = 1 \qquad 2x = 3$

$x = \frac{1}{2} \qquad x = \frac{3}{2}$

	$2x-1$	$2x-3$	V'	interval
$0 < x < \frac{1}{2}$	$-$	$-$	$+$	incr.
$\frac{1}{2} < x < \frac{3}{2}$	$+$	$-$	$-$	decr.
$x > \frac{3}{2}$	$+$	$+$	$+$	incr.

max at $x = \frac{1}{2}$

$V = (3 - 2\cdot\frac{1}{2})(3 - 2\cdot\frac{1}{2})\frac{1}{2}$

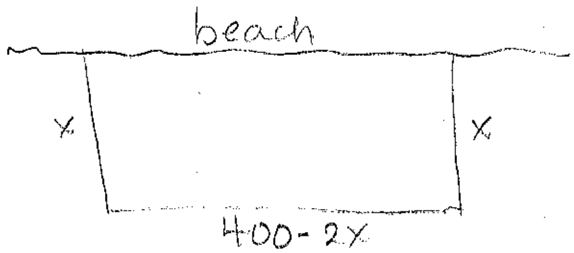
$= (3-1)(3-1)\frac{1}{2}$

$= 2 \cdot 2 \cdot \frac{1}{2}$

$= 2\ m^3$ largest volume

8-6 cont.

4.



a) $A = x(400-2x)$
$= 400x - 2x^2$
$A' = 400 - 4x$

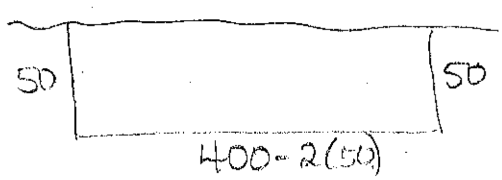
$400 - 4x = 0$
$400 = 4x$
$100 = x$

	$400-4x$	interval
$0 < x < 100$	+	incr.
$x > 100$	−	decr.

so $x = 100$ is a abs. max.

width = 100 m length = $400 - 2(100)$
$= 400 - 200$
$= 200$ m

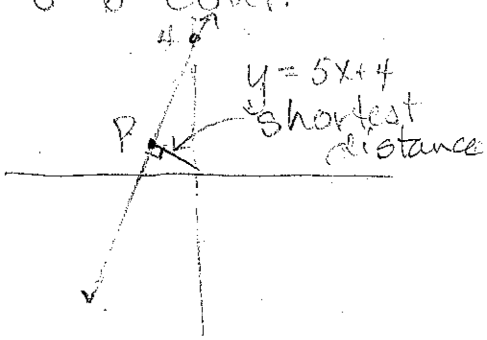
b)



width = 50 m length = $400 - 2(50)$
$= 400 - 100$
$= 300$ m

8-6 cont.

5.



Don't need calculus for this

Line of shortest distance is perpendicular with a y-intercept of 0 so equation is $y = -\frac{1}{5}x + 0$

$y = -\frac{1}{5}x$

Two equations - use substitution

$y = 5x+4$ $\qquad y = -\frac{1}{5}x$

$-\frac{1}{5}x = 5x+4$

$-x = 25x + 20$

$-26x = 20$

$x = -\frac{20}{26}$

$x = -\frac{10}{13}$

$y = 5\left(\frac{-10}{13}\right) + 4$

$y = \frac{-50}{13} + \frac{52}{13}$

$y = \frac{2}{13}$

$P = \left(\frac{-10}{13}, \frac{2}{13}\right)$