because the bracket is squared, pos. or neg. is irrelevant here

9-2 Vertical Asymptotes

1\. a) $\lim_{x\to 8} \frac{1}{(x-8)^2}$ As $x \to 8$ from either side, $(x-8)^2$ gets smaller. So $\frac{1}{(x-8)^2}$ gets larger.

limit is ∞ ← positive

b) $\lim_{x\to -1} \frac{-2}{(x+1)^2}$ as $x \to -1$ from either side, $(x+1)^2$ gets smaller. So $-\frac{2}{(x-8)^2}$ gets larger but negative

limit is $-\infty$

c) $\lim_{x\to 2^+} \frac{x-4}{x-2}$ as $x \to 2$ from the right, $x-2$ gets smaller positive and $x-4$ approaches -2 (because x is approaching 2). So $\frac{x-4}{x-2}$ $\left(\frac{neg}{pos}\right)$ gets larger negative.

limit is $-\infty$

d) $\lim_{x\to 2^-} \frac{x-4}{x-2}$ as $x \to 2$ from the left, $x-2$ gets smaller negative and $x-4$ approaches -2 (because x is approaching 2). So $\frac{x-4}{x-2}$ $\left(\frac{neg}{neg}\right)$ gets larger positive.

limit is ∞

e) $\lim_{x\to -4} \left[1 + \frac{2x}{(x+4)^6}\right]$ as $x \to -4$ from either side (because the bracket has exponent 6 an even exponent, pos. or neg. is irrelevant) $(x+4)^6$ gets smaller positive and $2x$ becomes negative. So $\frac{2x}{(x+4)^6}$ $\left(\frac{neg}{pos}\right)$ becomes larger negative and $1+\frac{2x}{(x+4)^6}$ becomes larger negative.

limit is $-\infty$

f) $\lim_{x\to -3^+} \frac{10}{x^2-x-12}$

$\lim_{x\to -3^+} \frac{10}{(x-4)(x+3)}$

as $x \to -3$ from the right $x-4$ approaches -7 and $x+3$ gets very small positive. So $(x-4)(x+3)$ is neg · pos = neg and $\frac{10}{(x-4)(x+3)}$ becomes very large negative

limit is $-\infty$

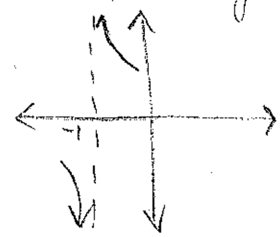
9.2 cont.

2. a) $y = \frac{2}{x+1}$

$x+1 \neq 0$
$x \neq -1$

$\lim_{x \to -1^+} \frac{2}{x+1}$ $\frac{\text{pos}}{\text{pos small}}$ = pos large ∞

$\lim_{x \to -1^-} \frac{2}{x+1}$ $\frac{\text{pos}}{\text{neg small}}$ = neg large $-\infty$

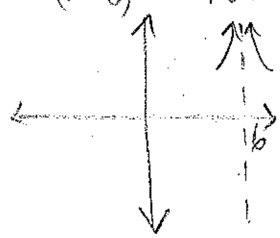


b) $y = \frac{3}{(x-6)^2}$

$x-6 \neq 0$
$x \neq 6$

$\lim_{x \to 6^+} \frac{3}{(x-6)^2}$ $\frac{\text{pos}}{\text{small pos}}$ = large pos ∞

$\lim_{x \to 6^-} \frac{3}{(x-6)^2}$ $\frac{\text{pos}}{\text{small pos}}$ = large pos ∞



c) $y = \frac{1}{x^2-1}$

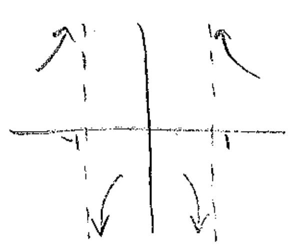
$x^2-1 \neq 0$
$x^2 \neq 1$
$x \neq \pm 1$

$\lim_{x \to 1^+} \frac{1}{x^2-1}$ $\frac{\text{pos}}{\text{small pos}}$ = large pos ∞

$\lim_{x \to 1^-} \frac{1}{x^2-1}$ $\frac{\text{pos}}{\text{small neg}}$ = large neg $-\infty$

$\lim_{x \to -1^+} \frac{1}{x^2-1}$ $\frac{\text{pos}}{\text{small neg}}$ = large neg $-\infty$

$\lim_{x \to -1^-} \frac{1}{x^2-1}$ $\frac{\text{pos}}{\text{small pos}}$ = large pos ∞



9.2 cont.

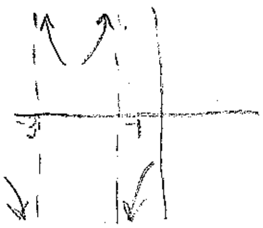
2. d) $y = \dfrac{6X^3}{X^2+4X+3}$

$X^2+4X+3 \neq 0$

$(X+3)(X+1) \neq 0$

$X+3 \neq 0,\ X+1 \neq 0$

$X = -3 \quad X \neq -1$



$\lim_{X \to -3^+} \dfrac{6X^3}{(X+3)(X+1)}$ $\dfrac{\text{neg}}{\text{pos small} \cdot \text{neg small}} = \dfrac{\text{neg}}{\text{neg small}} =$ pos large ∞

$\lim_{X \to -3^-} \dfrac{6X^3}{(X+3)(X+1)}$ $\dfrac{\text{neg}}{\text{neg small} \cdot \text{neg small}} = \dfrac{\text{neg}}{\text{pos small}} =$ neg large $-\infty$

$\lim_{X \to -1^+} \dfrac{6X^3}{(X+3)(X+1)}$ $\dfrac{\text{neg}}{\text{pos} \cdot \text{pos small}} = \dfrac{\text{neg}}{\text{pos small}} =$ neg large $-\infty$

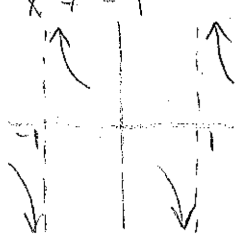
$\lim_{X \to -1^-} \dfrac{6X^3}{(X+3)(X+1)}$ $\dfrac{\text{neg}}{\text{pos} \cdot \text{neg small}} = \dfrac{\text{neg}}{\text{neg small}} =$ pos large ∞

e) $y = \dfrac{X}{X^2-1}$

$X^2-1 \neq 0$

$X^2 \neq 1$

$X \neq \pm 1$



$\lim_{X \to -1^+} \dfrac{X}{X^2-1}$ $\dfrac{\text{neg small}}{\text{neg smaller}} =$ pos large ∞

$\lim_{X \to -1^-} \dfrac{1}{X^2-1}$ $\dfrac{\text{neg small}}{\text{pos smaller}}$ neg large $-\infty$

$\lim_{X \to 1^+} \dfrac{X}{X^2-1}$ $\dfrac{\text{pos small}}{\text{pos smaller}} =$ pos large ∞

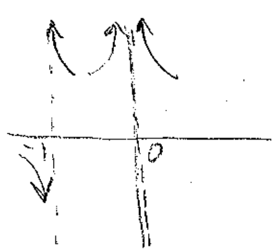
$\lim_{X \to 1^-} \dfrac{X}{X^2-1}$ $\dfrac{\text{pos small}}{\text{neg smaller}} =$ neg large $-\infty$

f) $y = \dfrac{1}{X^2(X+1)}$

$X^2(X+1) \neq 0$

$X = 0,\ X+1 \neq 0$

$X \neq -1$



$\lim_{X \to 0^+} \dfrac{1}{X^2(X+1)}$ $\dfrac{\text{pos}}{\text{pos} \cdot \text{pos}} = \dfrac{\text{pos}}{\text{pos small}} =$ pos large ∞

$\lim_{X \to 0^-} \dfrac{1}{X^2(X+1)}$ $\dfrac{\text{pos}}{\text{pos} \cdot \text{pos}} = \dfrac{\text{pos}}{\text{pos small}} =$ pos large ∞

$\lim_{X \to -1^+} \dfrac{1}{X^2(X+1)}$ $\dfrac{\text{pos}}{\text{pos} \cdot \text{pos}} = \dfrac{\text{pos}}{\text{pos small}} =$ pos large ∞

$\lim_{X \to -1^-} \dfrac{1}{X^2(X+1)}$ $\dfrac{\text{pos}}{\text{pos} \cdot \text{neg}} = \dfrac{\text{pos}}{\text{neg small}} =$ neg large $-\infty$