9-3 Horizontal Asymptotes

1. a) $\lim_{x \to -\infty} 3x^{-5} = \lim_{x \to -\infty} \frac{3}{x^5} = \lim_{x \to -\infty} 3 \cdot \frac{1}{x^5} = 3 \cdot 0 = 0$

b) $\lim_{x \to \infty} \frac{1-x}{3+5x} = \lim_{x \to \infty} \frac{\frac{1}{x} - 1}{\frac{3}{x} + 5} = \frac{0-1}{0+5} = \frac{-1}{5}$

c) $\lim_{x \to \infty} \frac{2x+1}{x-3} = \lim_{x \to \infty} \frac{2 + \frac{1}{x}}{1 - \frac{3}{x}} = \frac{2+0}{1-0} = \frac{2}{1} = 2$

d) $\lim_{x \to -\infty} \frac{2x+1}{x-3} = \lim_{x \to -\infty} \frac{2 + \frac{1}{x}}{1 - \frac{3}{x}} = \frac{2+0}{1-0} = \frac{2}{1} = 2$

e) $\lim_{x \to \infty} \frac{x^2-1}{(x+3)(2x+4)} = \lim_{x \to \infty} \frac{x^2-1}{2x^2+10x+12} = \lim_{x \to \infty} \frac{1 - \frac{1}{x^2}}{2 + \frac{10}{x} + \frac{12}{x^2}} =$
$\frac{1-0}{2+0+0} = \frac{1}{2}$

f) $\lim_{x \to -\infty} \frac{3x^3+x^2-5}{x^3-4x+1} = \lim_{x \to -\infty} \frac{3 + \frac{1}{x} - \frac{5}{x^3}}{1 + \frac{4}{x^2} + \frac{1}{x^3}} = \frac{3+0-0}{1+0+0} = \frac{3}{1} = 3$

2. a) $y = \frac{2x-3}{5-4x}$ $\lim_{x \to \infty} \frac{2x-3}{5-4x} = \lim_{x \to \infty} \frac{2 - \frac{3}{x}}{\frac{5}{x} - 4} = \frac{2-0}{0-4} = \frac{2}{-4} = \frac{-1}{2}$

hor. asymp. $y = \frac{-1}{2}$

b) $y = 1 - \frac{x}{x^2-2}$ $\lim_{x \to \infty} 1 - \frac{x}{x^2-2} = \lim_{x \to \infty} \frac{x^2-2}{x^2-2} - \frac{x}{x^2-2} =$
$\lim_{x \to \infty} \frac{x^2-x-2}{x^2-2} = \lim_{x \to \infty} \frac{1 - \frac{1}{x} - \frac{2}{x^2}}{1 - \frac{2}{x^2}} = \frac{1-0-0}{1-0} = \frac{1}{1} = 1$

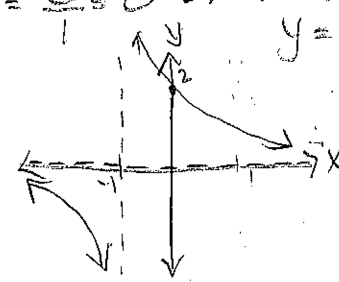
hor. asymp. $y = 1$

3. a) $y = \frac{2}{x+1}$ ① $\lim_{x \to \infty} \frac{2}{x+1} = \lim_{x \to \infty} \frac{\frac{2}{x}}{1 + \frac{1}{x}} = \frac{0}{1+0} = \frac{0}{1} = 0$ hor. asymp. $y = 0$

② $x+1 \neq 0$
$x \neq -1$
vert. asymp
$x = -1$

③ y-int
$y = \frac{2}{0+1}$
$y = \frac{2}{1} = 2$

④ x-int
$0 = \frac{2}{x+1}$
$0 = 2$
does not exist



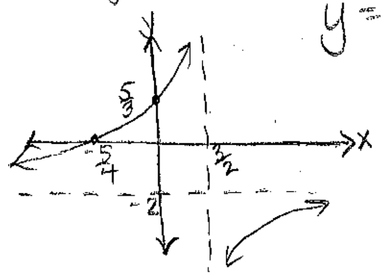
9-3 cont.

3. b) $y = \frac{4x+5}{3-2x}$ ① $\lim_{x \to \infty} \frac{4x+5}{3-2x} = \lim_{x\to\infty} \frac{4+\frac{5}{x}}{\frac{3}{x}-2} = \frac{4+0}{0-2} = \frac{4}{-2} = -2$ hor. asymp.

② $3-2x \neq 0$
$3 \neq 2x$
$\frac{3}{2} \neq x$
vert. asymp.

③ $y = \frac{4\cdot 0+5}{3-2\cdot 0}$
$y = \frac{0+5}{3-0}$
$y = \frac{5}{3}$ y-int

④ $0 = \frac{4x+5}{3-2x}$
$0 = 4x+5$
$-5 = 4x$
$-\frac{5}{4} = x$ x-int



c) $y = \frac{x}{x^2-1}$ ① $\lim_{x\to\infty} \frac{x}{x^2-1} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1-\frac{1}{x^2}} = \frac{0}{1-0} = \frac{0}{1} = 0$

② $x^2-1 \neq 0$
$x^2 \neq 1$
$x \neq \pm 1$
vert. asymp.

③ $y = \frac{0}{0^2-1}$
$y = 0$ y-int.

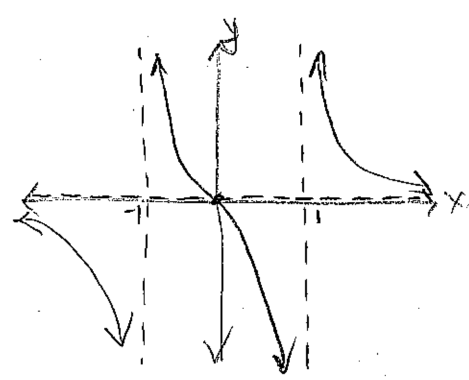
④ $0 = \frac{x}{x^2-1}$
$0 = x$ x-int

⑤ $\lim_{x\to -1^-} \frac{x}{x^2-1}$ $\frac{\text{neg}}{\text{pos small}}$ = neg large

$\lim_{x\to -1^+} \frac{x}{x^2-1}$ $\frac{\text{neg}}{\text{neg small}}$ = pos large

$\lim_{x\to 1^-} \frac{x}{x^2-1}$ $\frac{\text{pos}}{\text{neg small}}$ = neg large

$\lim_{x\to 1^+} \frac{x}{x^2-1}$ $\frac{\text{pos}}{\text{pos small}}$ = pos large



9.3 cont.

3. d) $y = \frac{2x^2}{x^2+3x-4}$

① $\lim_{x\to\infty} \frac{2x^2}{x^2+3x-4} = \lim_{x\to\infty} \frac{2}{1+\frac{3}{x}-\frac{4}{x^2}} = \frac{2}{1+0-0} = \frac{2}{1} = 2$ hor. asymp.

② $x^2+3x-4 \neq 0$
$(x+4)(x-1) \neq 0$
$x+4 \neq 0,\ x-1 \neq 0$
$x \neq -4 \quad x \neq 1$
vert. asymp.

③ $y = \frac{2\cdot 0^2}{0^2+3\cdot 0-4}$
$y = 0$
y-int.

④ $0 = \frac{2x^2}{x^2+3x-4}$
$0 = 2x^2$
$0 = x^2$
$0 = x$ x-int.

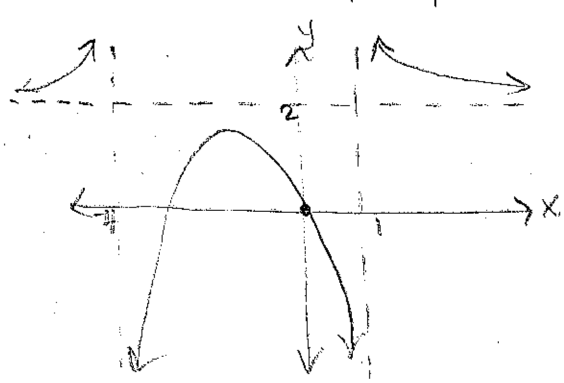
⑤ $y = \frac{2x^2}{x^2+3x-4} = \frac{2x^2}{(x+4)(x-1)}$

$\lim_{x\to -4^-} \frac{2x^2}{(x+4)(x-1)}$ $\frac{\text{pos}}{\text{neg neg small}} = \frac{\text{pos}}{\text{pos small}} = $ pos large

$\lim_{x\to -4^+} \frac{2x^2}{(x+4)(x-1)}$ $\frac{\text{pos}}{\text{pos neg}} = \frac{\text{pos}}{\text{neg small}} = $ neg large

$\lim_{x\to 1^-} \frac{2x^2}{(x+4)(x-1)}$ $\frac{\text{pos}}{\text{pos neg}} = \frac{\text{pos}}{\text{neg small}} = $ neg large

$\lim_{x\to 1^+} \frac{2x^2}{(x+4)(x-1)}$ $\frac{\text{pos}}{\text{pos pos}} = \frac{\text{pos}}{\text{pos small}} = $ pos large



9-3 cont.

4. a) $\lim_{x \to -\infty} x^5$

$= \lim_{x \to -\infty} x^4 \cdot x$

$= -\infty$

as $x \to -\infty$, x^4 becomes pos large and x becomes neg large so multiplied together it is neg large.

b) $\lim_{x \to \infty} (x^3 - x^2)$

$= \lim_{x \to \infty} x^2(x-1)$

$= \infty$

as $x \to \infty$, x^2 becomes pos large and $x-1$ becomes pos large so the result is pos large

c) $\lim_{x \to \infty} x^2(2x+1)(x-2)$

$= \infty$

as $x \to \infty$, x^2 become pos large, $(2x+1)$ becomes pos large, and $x-2$ becomes pos large so when all multiplied it's pos large

d) $\lim_{x \to \infty} (x+2)^4(3-x)$

$= -\infty$

as $x \to \infty$ $(x+2)^4$ becomeo pos large and $3-x$ becomes neg large so when multiplied these are neg larg