9-4 Concavity and Points of Inflection

1. a) $y = 2 + 5x - 12x^2$ concave down over $(-\infty, \infty)$

$y' = 5 - 24x$

$y'' = -24$

b) $y = 2x^3 + 24x^2 - 5x - 21$

$y' = 6x^2 + 48x - 5$

$y'' = 12x + 48$

$0 = 12x + 48$

$0 = 12(x + 4)$

$x + 4 = 0$

$x = -4$

$f(-4) = 2(-4)^3 + 24(-4)^2 - 5(-4) - 21$

$= -128 + 384 + 20 - 21$

$= 255$

inflection point $(-4, 255)$

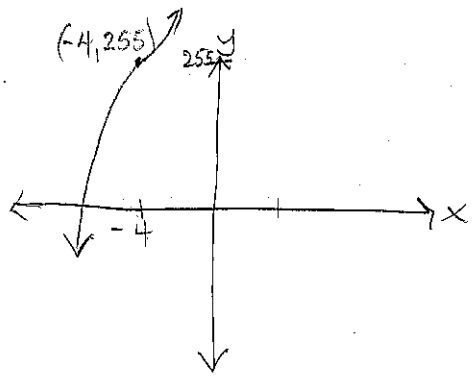
$12x + 48 > 0$

$12x > -48$

$x > -4$

Concave up $x > -4$

Concave down $x < -4$



c) $y = x^4 - 2x^3 + x - 2$

$y' = 4x^3 - 6x^2 + 1$

$y'' = 12x^2 - 12x$

$0 = 12x^2 - 12x$

$0 = 12x(x - 1)$

$x = 0$ or $x - 1 = 0$

$x = 1$

$f(0) = 0^4 - 2 \cdot 0^3 + 0 - 2$

$= -2$

$f(1) = 1^4 - 2(1)^3 + 1 - 2$

$= 1 - 2 + 1 - 2$

$= -2$

inflection points $(0, -2)$ $(1, -2)$

$12x^2 - 12x > 0$

$x^2 - x > 0$

$x(x - 1) > 0$

	x	$x-1$	y	result
$x < 0$	$-$	$-$	$+$	concave up
$0 < x < 1$	$+$	$-$	$-$	concave down
$x > 1$	$+$	$+$	$+$	concave up

concave up: $(-\infty, 0), (1, \infty)$

concave down: $(0, 1)$

9-4 cont.

1. d) $y = \frac{1}{x-1} = (x-1)^{-1}$

$y' = -(x-1)^{-2}(1) = -(x-1)^{-2}$

$y'' = -(-2)(x-1)^{-3} = 2(x-1)^{-3}$

	$x-1$	$f''(x)$	recall
$x<1$	−	−	con down
$x>1$	+	+	con up

concave up: $(1, \infty)$
concave down: $(-\infty, 1)$

$0 = \frac{2}{(x-1)^3}$

$0 = 2$

no inflection points

e) $y = \frac{1}{x^2+1} = (x^2+1)^{-1}$

$y' = -(x^2+1)^{-2}(2x)$

$= -2x(x^2+1)^{-2}$

$y'' = -2x(-2)(x^2+1)^{-3}(2x) + -2(x^2+1)^{-2}$

$= 8x^2(x^2+1)^{-3} - 2(x^2+1)^{-2}$

$= \frac{8x^2}{(x^2+1)^3} - \frac{2}{(x^2+1)^2}$

$= \frac{8x^2}{(x^2+1)^3} - \frac{2(x^2+1)}{(x^2+1)^3}$

$= \frac{8x^2 - 2x^2 - 2}{(x^2+1)^3}$

$= \frac{6x^2-2}{(x^2+1)^3}$

$(x^2+1)^3$ will always be positive

	$6x^2-2$	$f''(x)$	recall
$x < -\frac{1}{\sqrt{3}}$	+	+	con up
$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$	−	−	con down
$x > \frac{1}{\sqrt{3}}$	+	+	con up

concave up: $(-\infty, -\frac{1}{\sqrt{3}}), (\frac{1}{\sqrt{3}}, \infty)$
concave down: $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

$0 = \frac{6x^2-2}{(x^2+1)^3}$

$0 = 6x^2 - 2$

$2 = 6x^2$

$\frac{2}{6} = x^2$

$\frac{1}{3} = x^2$

$\pm\frac{1}{\sqrt{3}} = x$

$f(\frac{1}{\sqrt{3}}) = \frac{1}{(\frac{1}{\sqrt{3}})^2 + 1}$

$= \frac{1}{\frac{1}{3}+1}$

$= \frac{1}{\frac{4}{3}}$

$= \frac{3}{4}$

$f(-\frac{1}{\sqrt{3}}) = \frac{1}{(-\frac{1}{\sqrt{3}})^2 + 1}$

$= \frac{1}{\frac{1}{3}+1}$

$= \frac{1}{\frac{4}{3}}$

$= \frac{3}{4}$

inflection pts: $(\frac{1}{\sqrt{3}}, \frac{3}{4})$, $(-\frac{1}{\sqrt{3}}, \frac{3}{4})$

9-4 cont.

1. f) $y = \frac{1-x^2}{x^3}$ $\quad x \neq 0$

$= (1-x^2)x^{-3}$

$= x^{-3} - x^{-1}$

$y' = -3x^{-4} - (-1)x^{-2}$

$= -3x^{-4} + x^{-2}$

$y'' = 12x^{-5} + (-2)x^{-3}$

$= 12x^{-5} - 2x^{-3}$

$= \frac{12}{x^5} - \frac{2}{x^3}$

$= \frac{12}{x^5} - \frac{2x^2}{x^5}$

$= \frac{12-2x^2}{x^5}$

$0 = \frac{12-2x^2}{x^5}$

$0 = 12-2x^2$

$-12 = -2x^2$

$6 = x^2$

$\pm\sqrt{6} = x$

	$12-2x^2$	x^5	$f''(x)$	result
$x < -\sqrt{6}$	−	−	+	con up
$-\sqrt{6} < x < 0$	+	−	−	con down
$0 < x < \sqrt{6}$	+	+	+	con up
$x > \sqrt{6}$	−	+	−	con down

concave up: $(-\infty, -\sqrt{6})$, $(0, \sqrt{6})$

concave down: $(-\sqrt{6}, 0)$, $(\sqrt{6}, \infty)$

$f(\sqrt{6}) = \frac{1-\sqrt{6}^2}{\sqrt{6}^3}$

$= \frac{1-6}{6\sqrt{6}}$

$= \frac{-5}{6\sqrt{6}}$

$(\sqrt{6}, \frac{-5}{6\sqrt{6}})$

$f(-\sqrt{6}) = \frac{1-(-\sqrt{6})^2}{(-\sqrt{6})^3}$

$= \frac{1-6}{-6\sqrt{6}}$

$= \frac{-5}{-6\sqrt{6}}$

$= \frac{5}{6\sqrt{6}}$

$(-\sqrt{6}, \frac{5}{6\sqrt{6}})$

9.4 cont.

2. $y = 4 - 13x - 6x^2 - x^3$

a) $y' = -13 - 12x - 3x^2$

$0 = -13 - 12x - 3x^2$

$x = \frac{+12 \pm \sqrt{(-12)^2 - 4(-3)(-13)}}{2 \cdot 3}$

$x = \frac{+12 \pm \sqrt{144 - 156}}{6}$

$x = \frac{+12 \pm \sqrt{-12}}{6}$ no solution so no x-intercept

Substituting any value for x in y' gives a negative number so the function decreases over $(-\infty, \infty)$

b) Since it is decreasing over all intervals, there is no local minimum or maximum

c) $y'' = -12 - 6x$

$0 = -12 - 6x$

$12 = -6x$

$-2 = x$

	y''	result
$x < -2$	+	concave up
$x > -2$	-	concave down

concave up $(-\infty, -2)$

concave down $(-2, \infty)$

d) use $x = -2$ $\quad y = 4 - 13(-2) - 6(-2)^2 - (-2)^3$

$= 4 + 26 - 24 + 8$

$= 14$

inflection point $(-2, 14)$

e)