9-6 Sketching Curves

1. a) $y = 3x^5 - 10x^3 + 45x$

A. Domain: $\{x \mid x \in R\}$

B. Intercepts:

$y = 3 \cdot 0^5 - 10 \cdot 0^3 + 45 \cdot 0$
$y = 0 - 0 + 0$
$y = 0$

$0 = 3x^5 - 10x^3 + 45x$
$0 = x(3x^4 - 10x^2 + 45)$
$x^2 = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(45)}}{2 \cdot 3}$
$x^2 = \dfrac{10 \pm \sqrt{100 - 540}}{6}$
$x^2 = \dfrac{10 \pm \sqrt{-440}}{6}$ no solution

the only x-intercept can be $x = 0$

C. Symmetry:

$f(-x) = 3(-x)^5 - 10(-x)^3 + 45(-x)$
$= -3x^5 + 10x^3 - 45x$
$= -(3x^5 - 10x^3 + 45x) = -f(x)$

odd function → rotation symmetry

D. Asymptotes:

no denominator so no vertical asymptotes

$\lim_{x \to \infty} 3x^5 - 10x^3 + 45x$

as $x \to \infty$, each term gets very large so there is no horizontal asymptotes

E. Intervals of Increase or Decrease:

$f'(x) = 15x^4 - 30x^2 + 45$
$= 15(x^4 - 2x^2 + 3)$

$x^4 - 2x^2 + 3 > 0$
for all values of x so f(x) is increasing for all values of x.

9-6 cont.

1. a) cont.

F. Local max and min values:

Increasing over R, so no max or min

G. Concavity and points of inflection

$f''(x) = 60x^3 - 60x$

$= 60x(x^2-1)$

$0 = 60x(x^2-1)$

$x = 0, -1, 1$

	$60x$	x^2-1	$f''(x)$	f
$x<-1$	−	+	−	con down
$-1<x<0$	−	−	+	con. up
$0<x<1$	+	−	−	con down
$x>1$	+	+	+	con. up

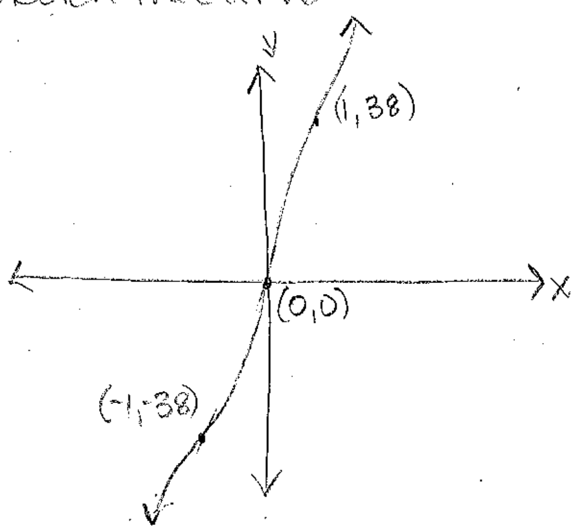
$y=3(0)^5-10(0)^3+45(0)$
$y=0-0+0$
$y=0$
$(0,0)$

$y=3(1)^5-10(1)^3+45(1)$
$y=3-10+45$
$y=38$
$(1,38)$

$y=3(-1)^5-10(-1)^3+45(-1)$
$y=-3+10-45$
$y=-38$
$(-1,-38)$

critical points

H. Sketch the curve



9-6 cont.

1. b) $y = (x^2-1)^3$

A. Domain: $\{x \mid x \in \mathbb{R}\}$

B. Intercepts:

$y = (0^2-1)^3$
$y = (-1)^3$
$y = -1$

$0 = (x^2-1)^3$
$0 = x^2-1$
$1 = x^2$
$x = 1$ or -1

C. Symmetry:

$f(-x) = ((-x)^2-1)^3$
$= (x^2-1)^3 = f(x)$

even function so reflection symmetry across y-axis

D. Asymptotes:

no vertical asymptotes because no denominator

$\lim_{x \to \infty} (x^2-1)^3$ as $x \to \infty$, the function gets very large so no horizontal asymptote

E. Intervals of Increase or Decrease:

$f'(x) = 3(x^2-1)^2(2x)$
$= 6x(x^2-1)^2$

$0 = 6x(x^2-1)^2$
$0 = 6x$, $(x^2-1)^2 = 0$
$0 = x$, $x^2-1 = 0$
$x^2 = 1$
$x = \pm 1$

critical numbers

	$6x$	$(x^2-1)^2$	$f'(x)$	
$x<-1$	$-$	$+$	$-$	dec
$-1<x<0$	$-$	$+$	$-$	dec
$0<x<1$	$+$	$+$	$+$	inc
$x>1$	$+$	$+$	$+$	inc

9-6 cont.

1. b) cont.

F. Local Max or Min values:

$f(0) = (0^2-1)^3$
$= (0-1)^3$
$= (-1)^3$
$= -1$

goes from dec to inc at $(0,-1)$ so it is a local min

min $= -1$

G. Concavity and Points of Inflection

$f''(x) = 6x(2)(x^2-1)(2x) + (6)(x^2-1)^2$
$= 24x^2(x^2-1) + 6(x^2-1)^2$
$= 6(x^2-1)[4x^2 + (x^2-1)]$
$= 6(x^2-1)(5x^2-1)$

$0 = 6(x^2-1)(5x^2-1)$

$x^2-1=0$, $5x^2-1=0$
$x^2=1$ $5x^2=1$
$x=\pm 1$ $x^2=\frac{1}{5}$

$x = \pm \frac{1}{\sqrt{5}}$

$\frac{1}{\sqrt{5}} \approx 0.45$

	x^2-1	$5x^2-1$	$f''(x)$	f
$x<-1$	+	+	+	con. up
$-1<x<-\frac{1}{\sqrt{5}}$	−	+	−	con. down
$-\frac{1}{\sqrt{5}}<x<\frac{1}{\sqrt{5}}$	−	−	+	con. up
$\frac{1}{\sqrt{5}}<x<1$	−	+	−	con. down
$x>1$	+	+	+	con. up

$f(-1) = ((-1)^2-1)^3 = (1-1)^3 = 0$ $(-1,0)$

$f(1) = (1^2-1)^3 = (1-1)^3 = 0$ $(1,0)$

$f(-\frac{1}{\sqrt{5}}) = ((-\frac{1}{\sqrt{5}})^2-1)^3 = (\frac{1}{5}-1)^3 = (-\frac{4}{5})^3 = -\frac{64}{125}$ $(-\frac{1}{\sqrt{5}}, \frac{-64}{125})$

$f(\frac{1}{\sqrt{5}}) = ((\frac{1}{\sqrt{5}})^2-1)^3 = (\frac{1}{5}-1)^3 = (-\frac{4}{5})^3 = -\frac{64}{125}$ $(\frac{1}{\sqrt{5}}, \frac{-64}{125})$

inflection points