9.7 Slant asymptotes

1. a) $y = \dfrac{2x - x^2 - 1}{x}$

$$x \overline{)\, -x^2 + 2x - 1} \quad -x + 2$$
$$-x^2$$
$$2x - 1$$
$$2x$$
$$-1$$

$y = -x + 2 - \dfrac{1}{x}$

$\lim_{x \to \infty} [f(x) - (-x+2)] =$

$\lim_{x \to \infty} \left[-x + 2 - \dfrac{1}{x} - (-x+2)\right] =$

$\lim_{x \to \infty} \left(-\dfrac{1}{x}\right) =$

0

$y = -x + 2$ is the slant asymptote

b) $y = \dfrac{x^3 - 1}{x^2}$

$$x^2 \overline{)\, x^3 - 1} \quad x$$
$$x^3$$
$$-1$$

$y = x - \dfrac{1}{x^2}$

$\lim_{x \to \infty} [f(x) - x] =$

$\lim_{x \to \infty} \left[x - \dfrac{1}{x^2} - x\right] =$

$\lim_{x \to \infty} \left(-\dfrac{1}{x^2}\right) =$

0

$y = x$ is the slant asymptote

c) $y = \dfrac{3x^2 + 4x + 2}{x + 1}$

$$x + 1 \overline{)\, 3x^2 + 4x + 2} \quad 3x + 1$$
$$3x^2 + 3x$$
$$x + 2$$
$$x + 1$$
$$1$$

$y = 3x + 1 + \dfrac{1}{x+1}$

$\lim_{x \to \infty} [f(x) - (3x+1)] =$

$\lim_{x \to \infty} \left[3x + 1 + \dfrac{1}{x+1} - (3x+1)\right] =$

$\lim_{x \to \infty} \dfrac{1}{x+1} =$

$\lim_{x \to \infty} \dfrac{\frac{1}{x}}{1 + \frac{1}{x}} = \dfrac{0}{1+0} = \dfrac{0}{1} = 0$

$y = 3x + 1$ is the slant asymptote

9-4 cont.

1. d) $y = \dfrac{x^3+4x^2+5x+16}{x^2+4}$

$$
\begin{array}{r}
x+4 \\
x^2+4 \overline{)\, x^3+4x^2+5x+16} \\
\underline{x^3 \quad +4x} \\
4x^2+x+16 \\
\underline{4x^2 \quad +16} \\
x
\end{array}
$$

$y = x+4+\dfrac{x}{x^2+4}$

$\lim\limits_{x\to\infty}\left[f(x)-(x+4)\right] =$

$\lim\limits_{x\to\infty}\left[x+4+\dfrac{x}{x^2+4}-(x+4)\right] =$

$\lim\limits_{x\to\infty}\dfrac{x}{x^2+4}$

$\lim\limits_{x\to\infty}\dfrac{\frac{1}{x}}{1+\frac{4}{x^2}} = \dfrac{0}{1+0} = 0$

$y = x+4$ is the slant asymptote