8.2 Pre-Calculus Math 11

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3. Substitution - I am going to verify the first one only!

a) $x^2 - y + 2 = 0$
$4x = 14 - y$

isolate 1 variable: $x^2 - y + 2 = 0$
$x^2 + 2 = y$

substitute into other equation and solve:

$4x = 14 - (x^2 + 2)$
$4x = 14 - x^2 - 2$
$4x = 12 - x^2$
$x^2 + 4x - 12 = 0$ can be factored
$(x + 6)(x - 2) = 0$
$x + 6 = 0$ or $x - 2 = 0$
$x = -6$ $x = 2$

substitute x values into one equation to find y-values

$4(-6) = 14 - y$
$-24 = 14 - y$
$-38 = -y$
$38 = y$

$4(2) = 14 - y$
$8 = 14 - y$
$-6 = -y$
$6 = y$

solutions: $(-6, 38)$ $(2, 6)$

verify each answer in <u>each</u> equation:

$(-6, 38)$ $(-6)^2 - 38 + 2 = 0$
$36 - 38 + 2 = 0$
$-2 + 2 = 0$
$0 = 0$

$(2, 6)$ $2^2 - 6 + 2 = 0$
$4 - 6 + 2 = 0$
$-2 + 2 = 0$
$0 = 0$

$4(-6) = 14 - 38$
$-24 = -24$

$4(2) = 14 - 6$
$8 = 8$

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3. b) $2x^2-4x+y=3$
$4x-2y=-7$

Solve for y: $2x^2-4x+y=3$
$y=-2x^2+4x+3$

Substitute into other equation and solve:

$4x-2(-2x^2+4x+3)=-7$
$4x+4x^2-8x-6=-7$
$4x^2-4x+1=0$ factor
$4x^2-2x-2x+1=0$
$2x(2x-1)-1(2x-1)=0$
$(2x-1)(2x-1)=0$
$2x-1=0$
$2x=1$
$x=\frac{1}{2}$ or 0.5

find y value:
$4(0.5)-2y=-7$
$2-2y=-7$
$2+7=2y$
$9=2y$
$4.5=y$

solution $(0.5, 4.5)$

c) $7d^2+5d-t-8=0$
$10d-2t=-40$

Solve for t: $7d^2+5d-t-8=0$
$7d^2+5d-8=t$

Substitute into other equation and solve:
$10d-2(7d^2+5d-8)=-40$
$10d-14d^2-10d+16=-40$
$-14d^2+16=-40$
$-14d^2=-56$
$d^2=4$
$d=\pm 2$

find t values:

$d=2$: $10(2)-2t=-40$
$20-2t=-40$
$-2t=-60$
$t=30$

$d=-2$: $10(-2)-2t=-40$
$-20-2t=-40$
$-2t=-20$
$t=10$

solutions $(2, 30)$ $(-2, 10)$

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3. d) $3x^2+4x-y-8=0$ solve for y: $3x^2+4x-y-8=0$

$y+3=2x^2+4x$ $3x^2+4x-8=y$

substitute y value into other equation and solve

$3x^2+4x-8+3=2x^2+4x$

$3x^2+4x-5=2x^2+4x$

$x^2-5=0$

$x^2=5$

$x=\pm\sqrt{5}$

$x=\pm 2.2361$

find y-values:

$x=+2.2361$

$y+3=2(2.2361)^2+4(2.2361)$

$y+3=2\cdot5+8.9443$

$y+3=10+8.9443$

$y+3=18.9443$

$y=15.9443$

$(2.24, 15.94)$

$x=-2.2361$

$y+3=2(-2.2361)^2+4(-2.2361)$

$y+3=2\cdot5-8.9443$

$y+3=10-8.9443$

$y+3=1.0557$

$y=-1.9443$

$(-2.24, -1.94)$

e) $y+2x=x^2-6$ solve for y: $y+2x=x^2-6$

$x+y-3=2x^2$ $y=x^2-2x-6$

substitute and solve:

$x+x^2-2x-6-3=2x^2$

$x^2-x-9=2x^2$

$0=x^2+x+9$ cannot factor

$a=1, b=1, c=9$

$x=\frac{-1\pm\sqrt{1^2-4(1)(9)}}{2(1)}$

$x=\frac{-1\pm\sqrt{1-36}}{2}$

$x=\frac{-1\pm\sqrt{-35}}{2}$ ← cannot take the square root of a negative number so no solution

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4. elimination - I will verify the first one

a) $6x^2 - 3x = 2y - 5$
$2x^2 + x = y - 4$ ← multiply this equation by 2 so you have 2y in both equations

$6x^2 - 3x = 2y - 5$
$4x^2 + 2x = 2y - 8$
$2x^2 - 5x = 3$ ← the 2y's are the same sign (pos) so subtract equations

Simplify and factor

$2x^2 - 5x = 3$
$2x^2 - 5x - 3 = 0$
$2x^2 - 6x + x - 3 = 0$
$2x(x-3) + 1(x-3) = 0$
$(x-3)(2x+1) = 0$
$x - 3 = 0$ or $2x + 1 = 0$
$x = 3$ $\quad 2x = -1$
$x = -\frac{1}{2}$

find y-values:

$2(3)^2 + 3 = y - 4$
$2 \cdot 9 + 3 = y - 4$
$18 + 3 = y - 4$
$21 = y - 4$
$25 = y$

$2\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) = y - 4$
$2 \cdot \frac{1}{4} - \frac{1}{2} = y - 4$
$0 = y - 4$
$4 = y$

Solutions: $(3, 25)$ $\left(-\frac{1}{2}, 4\right)$

verify: $(3, 25)$
$6(3)^2 - 3(3) = 2(25) - 5$
$6 \cdot 9 - 9 = 50 - 5$
$54 - 9 = 45$
$45 = 45$

$2(3)^2 + 3 = 25 - 4$
$2 \cdot 9 + 3 = 21$
$18 + 3 = 21$
$21 = 21$

$\left(-\frac{1}{2}, 5\right)$
$6\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) = 2(4) - 5$
$6 \cdot \frac{1}{4} + \frac{3}{2} = 8 - 5$
$\frac{6}{2} = 3$
$3 = 3$

$2\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) = 4 - 4$
$2\left(\frac{1}{4}\right) - \frac{1}{2} = 0$
$0 = 0$

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4. b) $x^2 + y = 8x + 19$
$x^2 - y = 7x - 11$
$2x^2 = 15x + 8$

Just add because the y's already have the same coefficient

$2x^2 = 15x + 8$ ← solve
$2x^2 - 15x - 8 = 0$
$2x^2 - 16x + x - 8 = 0$
$2x(x-8) + 1(x-8) = 0$
$(x-8)(2x+1) = 0$
$x - 8 = 0$ or $2x + 1 = 0$
$x = 8$ $\quad 2x = -1$
$x = -\frac{1}{2}$

find y-values

$x = 8$
$8^2 + y = 8(8) + 19$
$64 + y = 64 + 19$
$y = 19$

$x = -\frac{1}{2}$
$(-\frac{1}{2})^2 + y = 8(-\frac{1}{2}) + 19$
$\frac{1}{4} + y = -4 + 19$
$\frac{1}{4} + y = 15$
$y = 15 - \frac{1}{4}$
$y = 14.75$

solutions: $(8, 19)$ $(-\frac{1}{2}, 14.75)$

c) $2p^2 = 4p - 2m + 6$
$5m + 8 = 10p + 5p^2$

rearrange so the terms are in the same order in each equation

$2p^2 - 4p = -2m + 6$ ← multiply by 5
$5p^2 + 10p = 5m + 8$ ← multiply by 2
to get $-10m$ and $10m$

$10p^2 - 20p = -10m + 30$
$10p^2 + 20p = 10m + 16$
$20p^2 = 46$

add because you have $-10m$ and $10m$

$p^2 = \frac{46}{20}$
$p^2 = 2.3$
$p = \pm 1.5166$

find m values:

$p = 1.5166$
$2(1.5166)^2 = 4(1.5166) - 2m + 6$
$2(2.3) = 6.0664 - 2m + 6$
$4.6 = 12.0664 - 2m$
$-7.4664 = -2m$
$3.7332 = m$

$p = -1.5166$
$2(-1.5166)^2 = 4(-1.5166) - 2m + 6$
$2(2.3) = -6.0664 - 2m + 6$
$4.6 = -0.0664 - 2m$
$4.6664 = -2m$
$-2.3332 = m$

solutions: $(1.52, 3.73)$
$(-1.52, -2.33)$

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4. d) $9w^2 + 8k = -14$
$w^2 + k = -2$ ← multiply by 9

$9w^2 + 8k = -14$
$9w^2 + 9k = -18$ subtract
$-k = 4$
$k = -4$

find w values: $w^2 + -4 = -2$
$w^2 = 2$
$w = \pm 1.4142$

solutions
$(1.41, -4)$ $(-1.41, -4)$

The one value for k gives two values for w so the -4 is for both solutions

e) $4h^2 - 8t = 6$
$6h^2 - 9 = 12t$ put in the same order

$4h^2 - 8t = 6$ ← multiply by 3
$6h^2 - 12t = 9$ ← multiply by 2

$12h^2 - 24t = 18$
$12h^2 - 24t = 18$

These are exactly the same equation so there will be an infinite number of solutions.

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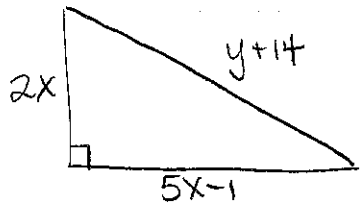


perimeter = 60
area = 10y

a) perimeter expression
$2x + 5x - 1 + y + 14$
$7x + y + 13$

b) area expression
$\frac{2x(5x-1)}{2} = 5x^2 - x$

c) $7x + y + 13 = 60$ perimeter equation
$5x^2 - x = 10y$ area equation

d) $7x + y + 13 = 60$
$y = 60 - 7x - 13$
$y = 47 - 7x$

$5x^2 - x = 10(47 - 7x)$
$5x^2 - x = 470 - 70x$
$5x^2 + 69x - 470 = 0$
$a = 5$, $b = 69$, $c = -470$

$x = \frac{-69 \pm \sqrt{69^2 - 4(5)(-470)}}{2(5)}$

$x = \frac{-69 \pm \sqrt{4761 + 9400}}{10}$

$x = \frac{-69 \pm \sqrt{14161}}{10}$

$x = \frac{-69 \pm 119}{10}$

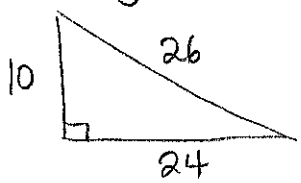
$x = \frac{-69 + 119}{10}$ or $\frac{-69 - 119}{10}$

$x = 5$ $\quad \frac{-188}{10}$

-18.8

length can't be negative

find y value
$y = 47 - 7 \cdot 5$
$y = 47 - 35$
$y = 12$



e) verify:
$10 + 26 + 24 = 60$

$\frac{5 \cdot 10 \cdot 24}{2} = 10(12)$

$120 = 120$