Pre-Calculus Math 11

page 193 #6, 7, 12-14, 18

6. a) $y = x^2 + 6x - 2$
$y = (x^2 + 6x + 9 - 9) - 2$
$y = (x^2 + 6x + 9) - 9 - 2$
$y = (x+3)^2 - 11$

vertex $(-3, -11)$
Opens up ($+1$ in front of x^2)
min of -11 when $x = -3$

b) $y = 3x^2 - 12x + 1$
$y = 3(x^2 - 4x) + 1$
$y = 3(x^2 - 4x + 4 - 4) + 1$
$y = 3(x^2 - 4x + 4) - 12 + 1$
$y = 3(x-2)^2 - 11$

vertex $(2, -11)$
Opens up ($+3$ with the x^2)
min of -11 when $x = 2$

c) $y = -x^2 - 10x$
$y = -(x^2 + 10x)$
$y = -(x^2 + 10x + 25 - 25)$
$y = -(x^2 + 10x + 25) + 25$
$y = -(x+5)^2 + 25$

vertex $(-5, 25)$
Opens down (-1 with the x^2)
max of 25 when $x = -5$

d) $y = -2x^2 + 8x - 3$
$y = -2(x^2 - 4x) - 3$
$y = -2(x^2 - 4x + 4 - 4) - 3$
$y = -2(x^2 - 4x + 4) + 8 - 3$
$y = -2(x-2)^2 + 5$

vertex $(2, 5)$
opens down (-2 with the x^2
max of 5 when $x = 2$

$\frac{1}{2}$ of 5 is $\frac{5}{2}$, then $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

7. a) $f(x) = x^2 + 5x + 3$
$f(x) = (x^2 + 5x + \frac{25}{4} - \frac{25}{4}) + 3$
$f(x) = (x^2 + 5x + \frac{25}{4}) - \frac{25}{4} + \frac{12}{4}$
$f(x) = (x + \frac{5}{2})^2 - \frac{13}{14}$

$\frac{3}{1} = \frac{12}{4}$

vertex $\left(-\frac{5}{2}, -\frac{13}{14}\right)$
opens up
min: $-\frac{13}{14}$

b) $f(x) = 2x^2 - 2x + 1$
$f(x) = 2(x^2 - x) + 1$
$f(x) = 2(x^2 - x + \frac{1}{4} - \frac{1}{4}) + 1$
$f(x) = 2(x^2 - x + \frac{1}{4}) - \frac{1}{2} + 1$
$f(x) = 2(x - \frac{1}{2})^2 + \frac{1}{2}$

$\frac{1}{2}$ of 1 is $\frac{1}{2}$, then $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

vertex $\left(\frac{1}{2}, \frac{1}{2}\right)$
Opens up
min: $\frac{1}{2}$

page 193 cont.

7. c) $f(x) = -0.5x^2 + 10x - 3$ $\quad \frac{10x}{0.5} = 20x$!!

$f(x) = -0.5(x^2 - 20x) - 3$

$f(x) = -0.5(x^2 - 20x + 100 - 100) - 3$ $\quad$ vertex $(10, 47)$

$f(x) = -0.5(x^2 - 20x + 100) + 50 - 3$ $\quad$ opens down

$f(x) = -0.5(x - 10)^2 + 47$ $\quad$ max 47

d) $f(x) = 3x^2 - 4.8x$ $\quad$ vertex $(0.8, -1.92)$

$f(x) = 3(x^2 - 1.6x)$ $\quad$ opens up

$f(x) = 3(x^2 - 1.6x + 0.64 - 0.64)$ $\quad$ min -1.92

$f(x) = 3(x^2 - 1.6x + 0.64) - 1.92$

$f(x) = 3(x - 0.8)^2 - 1.92$

e) $f(x) = -0.2x^2 + 3.4x + 4.5$ $\quad$ vertex $(8.5, 18.95)$

$f(x) = -0.2(x^2 - 17x) + 4.5$ $\quad$ opens down

$f(x) = -0.2(x^2 - 17x + 72.25 - 72.25) + 4.5$ $\quad$ max 18.95

$f(x) = -0.2(x^2 - 17x + 72.25) + 14.45 + 4.5$

$f(x) = -0.2(x - 8.5)^2 + 18.95$

f) $f(x) = -2x^2 + 5.8x - 3$ $\quad$ vertex $(1.45, 1.205)$

$f(x) = -2(x^2 - 2.9x) - 3$ $\quad$ opens down

$f(x) = -2(x^2 - 2.9x + 2.1025 - 2.1025) - 3$ $\quad$ max 1.205

$f(x) = -2(x^2 - 2.9x + 2.1025) + 4.205 - 3$

$f(x) = -2(x - 1.45)^2 + 1.205$

12. a) $y = x^2 + 8x + 30$

$y = (x^2 + \underline{8x + 16 - 16}) + 30$ $\quad$ error corrected here

$y = (x^2 + 8x + 16) - 16 + 30$

$y = (x + 4)^2 + 14$

b) $f(x) = 2x^2 - 9x - 55$

$f(x) = 2(x^2 - 4.5x) - 55$ $\quad$ error corrected here

$f(x) = 2(x^2 - 4.5x + \underline{5.0625 - 5.0625}) - 55$

$f(x) = 2(x^2 - 4.5x + 5.0625) - 10.125 - 55$

$f(x) = 2(x - 2.25)^2 - 65.125$

page 193 cont.

12\. c)
$$y = 8x^2 + 16x - 13$$
$$y = 8(x^2 + 2x) - 13$$
$$y = 8(x^2 + 2x + 1 - 1) - 13$$ error corrected here
$$y = 8(x^2 + 2x + 1) - 8 - 13$$
$$y = 8(x+1)^2 - 21$$

d)
$$f(x) = -3x^2 - 6x$$
$$f(x) = -3(x^2 - 2x)$$ error corrected here
$$f(x) = -3(x^2 - 2x + 1 - 1)$$
$$f(x) = -3(x^2 - 2x + 1) + 3$$
$$f(x) = -3(x-1)^2 + 3$$

13\.
$$C(n) = 75n^2 - 1800n + 60000$$
$$C(n) = 75(n^2 - 24n) + 60000$$
$$C(n) = 75(n^2 - 24n + 144 - 144) + 60000$$
$$C(n) = 75(n^2 - 24n + 144) - 10800 + 60000$$
$$C(n) = 75(n-12)^2 + 49\,200$$

vertex $(12, 49200)$ 12 is 12 thousand that is the number of items that will minimize costs

14\.
$$h(t) = -5t^2 + 10t + 4$$
$$h(t) = -5(t^2 - 2t) + 4$$
$$h(t) = -5(t^2 - 2t + 1 - 1) + 4$$
$$h(t) = -5(t^2 - 2t + 1) + 5 + 4$$
$$h(t) = -5(t-1)^2 + 9$$

vertex $(1, 9)$ max height is 9 metres.

page 193 cont.

18. \$70 per ticket → sold 2000 tickets
every \$1 decrease → sell 50 more tickets

Revenue = ticket price × number of tickets sold

$R(n) = (70-n)(2000+50n)$

reduce the price by "n" dollars

then the number of tickets sold goes up 50 for each dollar reduced.

$R(n) = (70-n)(2000+50n)$
$R(n) = 140000 + 3500n - 2000n - 50n^2$
$R(n) = -50n^2 + 1500n + 140000$
$R(n) = -50(n^2 - 30n) + 140000$
$R(n) = -50(n^2 - 30n + 225 - 225) + 140000$
$R(n) = -50(n^2 - 30n + 225) + 11250 + 140000$
$R(n) = -50(n-15)^2 + 151250$

opens down   vertex $(15, 151250)$

a) maximum revenue: \$151 250
this maximum occurs when the ticket price has been reduced by \$15 so the ticket price is $70 - 15 = \$55$

b) $\frac{\$151250}{\$55 \text{ per ticket}} = 2750$ tickets

c) assumptions
- the information provided is accurate
- the change in ticket price results in a consistent change in tickets sold.