7.3 Pre-Calculus Math 11

page 389 #4, 6, 12

4. a) $|x+7| = 12$

case 1: positive
restriction: $x+7 \geq 0$
$x \geq -7$
solve: $x+7 = 12$
$x = 5$
solution (because $5 \geq -7$)

case 2: negative
restriction: $x+7 < 0$
$x < -7$
solve: $-(x+7) = 12$
$-x-7 = 12$
$-x = 19$
$x = -19$
solution (because $-19 < -7$)

b) $|3x-4| + 5 = 7$
$|3x-4| = 2$

case 1: positive
restriction: $3x-4 \geq 0$
$3x \geq 4$
$x \geq \frac{4}{3}$

solve: $3x-4 = 2$
$3x = 6$
$x = 2$
solution

case 2: negative
restriction: $3x-4 < 0$
$3x < 4$
$x < \frac{4}{3}$

solve: $-(3x-4) = 2$
$-3x + 4 = 2$
$-3x = -2$
$x = \frac{2}{3}$
solution

c) $2|x+6| + 12 = -4$
$2|x+6| = -16$
$|x+6| = -8$

no solution

cannot be solved because the absolute value cannot be equal to a negative number

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4. d) $-6|2x-14| = -42$

$|2x-14| = 7$

case 1: positive

restriction: $2x-14 \geq 0$

$2x \geq 14$

$x \geq 7$

solve: $2x-14=7$

$2x = 21$

$x = 10.5$

solution

case 2: negative

restriction: $2x-14<0$

$2x < 14$

$x < 7$

solve: $-(2x-14) = 7$

$-2x+14 = 7$

$-2x = -7$

$x = 3.5$

solution

6. a) $|x| = x^2+x-3$

case 1: positive

restriction: $x \geq 0$

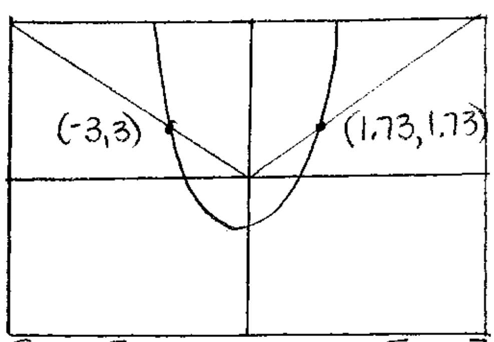
solve: $x = x^2+x-3$

$0 = x^2-3$

$3 = x^2$

$\pm\sqrt{3} = x$

$x=\sqrt{3}$ is the only solution because $x \geq 0$



[-10,10] x min max

[-10,10] y min max

case 2: negative

restriction: $x<0$

solve: $-x = x^2+x-3$

$0 = x^2+2x-3$

$0 = (x+3)(x-1)$

$x+3=0$ or $x-1=0$

$x=-3$ solution

$x=1$ not a solution because $x<0$

$\sqrt{3} = 1.73$

the two x-values are: -3 and $\sqrt{3}$

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6. b) $|x^2-2x+2| = 3x-4$

case 1: positive

restriction: $x^2-2x+2 \geq 0$

find the x-intercepts:

$(x^2-2x+1-1)+2$

$(x^2-2x+1)-1+2$

$(x-1)^2+1$

vertex $(1,1)$

opens up

there is no x-intercepts so all values of x will be ≥ 0

no restriction!

case 2: negative

restriction: $x^2-2x+2<0$

no values of x will allow this to be <0 so there is no solution

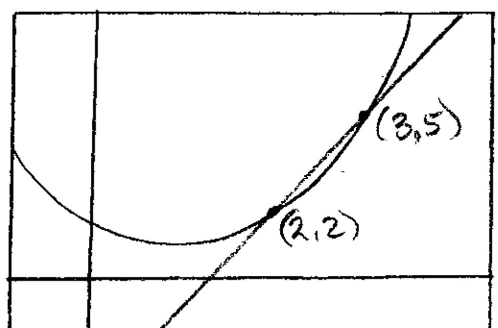
solve: $x^2-2x+2=3x-4$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$x-2=0$ or $x-3=0$

$x=2$ $\quad$ $x=3$



$[-1,5]$ x min max $\quad$ $[-2,10]$ y min max

c) $|x^2-9| = x^2-9$

case 1: positive

restriction: $x^2-9 \geq 0$

zeros are ± 3 so

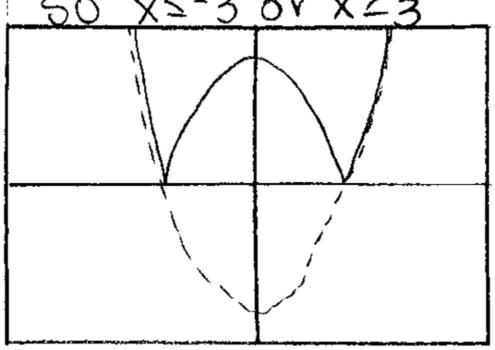
$x^2-9 \geq 0$ when $x \leq -3$

or $x \geq 3$

solve: $x^2-9=x^2-9$

for all values of x

so $x \leq -3$ or $x \geq 3$



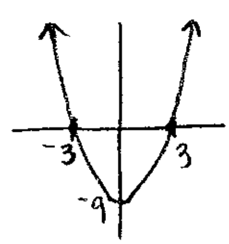
$---$ $y=x^2-9$

$—$ $y=|x^2-9|$

case 2: negative

restriction: $x^2-9<0$

$-3<x<3$



positive $y>0$

negative $y<0$

$y=x^2-9$

solve: $-(x^2-9)=x^2-9$

$-x^2+9=x^2-9$

$9=-9$

no solution

overlap is $x \leq -3$ or $x \geq 3$

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6. d) $|x^2-1| = x$

case 1: positive

rest: $x^2-1 \geq 0$

zeros are ± 1 so

$x \leq -1$ or $x \geq 1$

solve: $x^2-1=x$

$x^2-x-1=0$

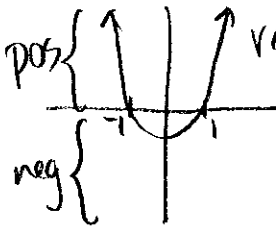
$x = \dfrac{-(-1) \pm \sqrt{(-1)^2-4(1)(-1)}}{2(1)}$

$x = \dfrac{1 \pm \sqrt{1+4}}{2}$

$x = \dfrac{1 \pm \sqrt{5}}{2}$

$x =$ 1.62 or -0.62

solution | not a solution



case 2: negative

rest: $-1 < x < 1$

solve: $-(x^2-1) = x$

$-x^2+1 = x$

$0 = x^2+x-1$

$x = \dfrac{-1 \pm \sqrt{1^2-4(1)(-1)}}{2(1)}$

$x = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$x = \dfrac{-1 \pm \sqrt{5}}{2}$

$x =$ -1.62 or 0.62

not a solution | solution

e) $|x^2-2x-16| = 8$

case 1: positive

restriction: $x^2-2x-16 \geq 0$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2-4(1)(-16)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{4+64}}{2}$

$x = \dfrac{2 \pm \sqrt{68}}{2}$

zero's are: 5.12 and -3.12

so restriction is: $x \leq -3.12$ or $x > 5.12$

solve: $x^2-2x-16=8$

$x^2-2x-24=0$

$(x-6)(x+4)=0$

$x-6=0$ or $x+4=0$

$x=6$ $\quad$ $x=-4$

both are solutions

case 2: negative

restriction:

$-3.12 < x < 5.12$

solve: $-(x^2-2x-16)=8$

$-x^2+2x+16=8$

$0=x^2-2x-8$

$0=(x-4)(x+2)$

$x-4=0$ or $x+2=0$

$x=4$ $\quad$ $x=-2$

both are solutions

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12. $|d-25150| = 381\,550$

a) case 1: positive

restriction: $d-25150 \geq 0$

$d \geq 25150$

solve: $d-25150 = 381\,550$

$d = 406\,700$

solution

case 2: negative

restriction: $d-25150 < 0$

$d < 25\,150$

solve: $-(d-25150) = 381\,550$

$-d + 25150 = 381550$

$-d = 356\,400$

$d = -356\,400$

solution

b) The average distance from the moon to the earth is 381 550 km but it can be up to 25 150 km closer or farther away.