

9.2 Pre-Calculus Math II

page 484 #4-7

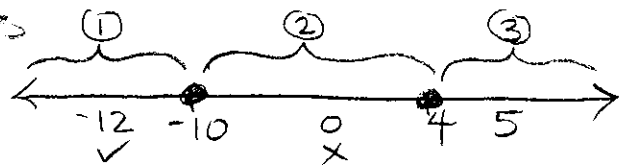
4. roots and test points

a) $x(x+6) \geq 40$

$$x^2 + 6x - 40 = 0$$

$$(x+10)(x-4) = 0$$

$$x = -10 \text{ or } 4$$



① $-12(-12+6) \geq 40$

$$-12(-6) \geq 40$$

$$72 \geq 40$$

true

② $0(0+6) \geq 40$

$$0(6) \geq 40$$

$$0 \geq 40$$

not true

③ $5(5+6) \geq 40$

$$5(11) \geq 40$$

$$55 \geq 40$$

true

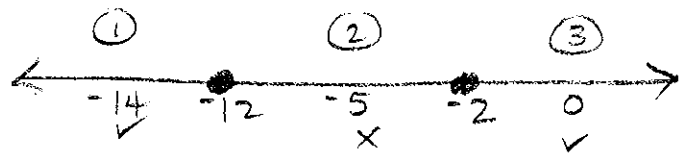
$$\{x \mid x \leq -10 \text{ or } x \geq 4, x \in \mathbb{R}\}$$

b) $-x^2 - 14x - 24 \leq 0$

$$-(x^2 + 14x + 24) \leq 0$$

$$-(x+12)(x+2) \leq 0$$

$$x = -12 \text{ or } -2$$



① $-(-14)^2 - 14(-14) - 24 \leq 0$

$$-196 - 196 - 24 \leq 0$$

$$-416 \leq 0$$

true

② $-(-5)^2 - 14(-5) - 24 \leq 0$

$$-25 + 70 - 24 \leq 0$$

$$21 \leq 0$$

not true

③ $-0^2 - 14(0) - 24 \leq 0$

$$0 - 0 - 24 \leq 0$$

$$-24 \leq 0$$

true

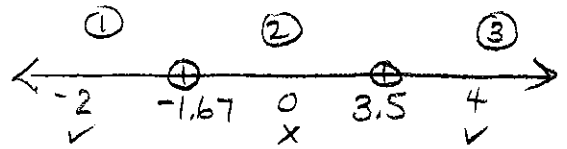
$$\{x \mid x \leq -12 \text{ or } x \geq -2, x \in \mathbb{R}\}$$

page 484 cont.

4.

$$\begin{aligned}
 c) \quad & 6x^2 > 11x + 35 \\
 & 6x^2 - 11x - 35 > 0 \\
 & 6x^2 - 21x + 10x - 35 = 0 \\
 & 3x(2x-7) + 5(2x-7) = 0 \\
 & (2x-7)(3x+5) = 0 \\
 & 2x-7=0 \text{ or } 3x+5=0 \\
 & 2x=7 \qquad 3x=-5 \\
 & x = \frac{7}{2} \text{ or } 3.5 \qquad x = -\frac{5}{3} \text{ or } -1.67
 \end{aligned}$$

$$\begin{aligned}
 & -(35 \cdot 6) \\
 & -(5 \cdot 7 \cdot 2 \cdot 3) \qquad -21, 10
 \end{aligned}$$



$$\begin{aligned}
 \textcircled{1} \quad & 6(-2)^2 > 11(-2) + 35 \\
 & 24 > -22 + 35 \\
 & 24 > 13 \\
 & \text{true}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad & 6(0)^2 > 11(0) + 35 \\
 & 0 > 0 + 35 \\
 & 0 > 35 \\
 & \text{not true}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad & 6(4)^2 > 11(4) + 35 \\
 & 96 > 44 + 35 \\
 & 96 > 79 \\
 & \text{true}
 \end{aligned}$$

$$\left\{ x \mid x < -\frac{5}{3} \text{ or } x > \frac{7}{2}, x \in \mathbb{R} \right\}$$

use the exact value, not approx. decimal

$$\begin{aligned}
 d) \quad & 8x + 5 \leq -2x^2 \\
 & 2x^2 + 8x + 5 \leq 0 \\
 & a=2, b=8, c=5
 \end{aligned}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64 - 40}}{4}$$

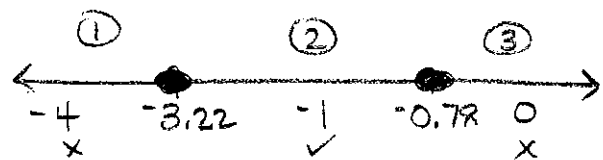
$$x = \frac{-8 \pm \sqrt{24}}{4}$$

$$x = \frac{-8 \pm 2\sqrt{6}}{4}$$

$$x = \frac{-4 \pm \sqrt{6}}{2}$$

$$x = \frac{-4 + \sqrt{6}}{2} \text{ or } \frac{-4 - \sqrt{6}}{2}$$

$$x = -0.78 \text{ or } -3.22$$



$$\begin{aligned}
 \textcircled{1} \quad & 8(-4) + 5 \leq -2(-4)^2 \\
 & -32 + 5 \leq -32 \\
 & -27 \leq -32 \\
 & \text{not true}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad & 8(-1) + 5 \leq -2(-1)^2 \\
 & -8 + 5 \leq -2 \\
 & -3 \leq -2 \\
 & \text{true}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad & 8(0) + 5 \leq -2(0)^2 \\
 & 5 \leq 0 \\
 & \text{not true}
 \end{aligned}$$

$$\left\{ x \mid \frac{-4 - \sqrt{6}}{2} \leq x \leq \frac{-4 + \sqrt{6}}{2}, x \in \mathbb{R} \right\}$$

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5. sign analysis

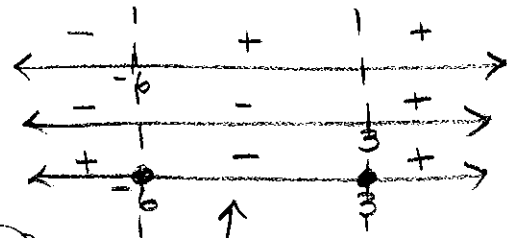
a) $x^2 + 3x \leq 18$

$x^2 + 3x - 18 \leq 0$

$(x+6)(x-3) = 0$

$x = -6 \text{ or } 3$

$x+6$
 $x-3$
 $(x+6)(x-3)$



less than zero means "negative"

$\{x \mid -6 \leq x \leq 3, x \in \mathbb{R}\}$

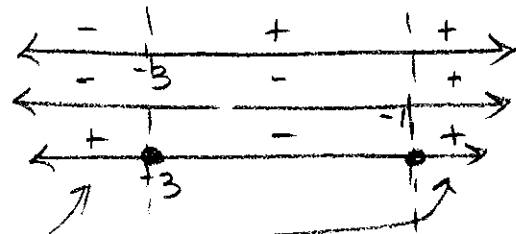
b) $x^2 + 3 \geq -4x$

$x^2 + 4x + 3 \geq 0$

$(x+3)(x+1) \geq 0$

$x = -3 \text{ or } -1$

$x+3$
 $x+1$
 $(x+3)(x+1)$



positive

$\{x \mid x \leq -3 \text{ or } x \geq -1, x \in \mathbb{R}\}$

c) $4x^2 - 27x + 18 < 0$

$4x^2 - 24x - 3x + 18 < 0$

$4x(x-6) - 3(x-6) < 0$

$(x-6)(4x-3) < 0$

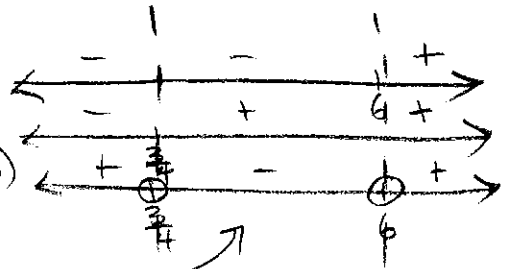
$x-6=0 \text{ or } 4x-3=0$

$x=6$

$4x=3$

$x = \frac{3}{4}$

$x-6$
 $4x-3$
 $(x-6)(4x-3)$



negative

$\{x \mid \frac{3}{4} < x < 6, x \in \mathbb{R}\}$

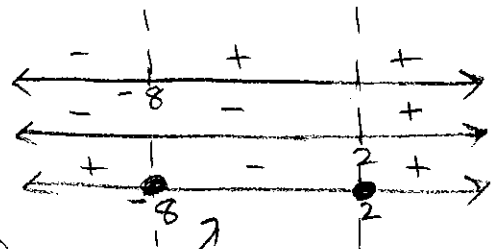
d) $-6x \geq x^2 - 16$

$0 \geq x^2 + 6x - 16$

$0 \geq (x+8)(x-2)$

$x = -8 \text{ or } 2$

$x+8$
 $x-2$
 $(x+8)(x-2)$



negative because zero is bigger

$\{x \mid -8 \leq x \leq 2, x \in \mathbb{R}\}$

2.2.3.3.2
24.3

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6. Case analysis

a) $x^2 - 2x - 15 < 0$
 $(x-5)(x+3) < 0$

negative

one bracket positive and one is negative

Case 1: $x-5 \geq 0$

$x \geq 5$

$x+3 \leq 0$

$x \leq -3$



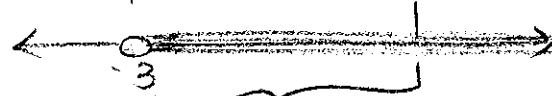
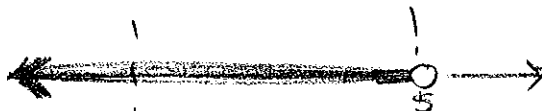
no solution

Case 2: $x-5 \leq 0$

$x \leq 5$

$x+3 \geq 0$

$x \geq -3$



Overlap

$\{x \mid -3 < x < 5, x \in \mathbb{R}\}$

b) $x^2 + 13x > -12$

$x^2 + 13x + 12 > 0$

$(x+12)(x+1) > 0$

positive

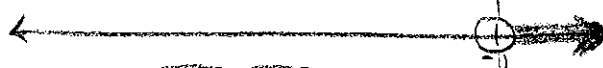
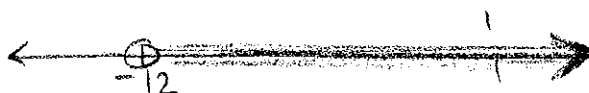
both brackets are positive or both negative

Case 1: $x+12 > 0$

$x > -12$

$x+1 > 0$

$x > -1$



Overlap

Case 2: $x+12 < 0$

$x < -12$

$x+1 < 0$

$x < -1$



Overlap

$\{x \mid x < -12 \text{ or } x > -1, x \in \mathbb{R}\}$

page 484 cont.
 6. c) $-x^2 + 2x + 5 \leq 0$
 $a = -1, b = 2, c = 5$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(-1)(5)}}{2(-1)}$$

$$x = \frac{-2 \pm \sqrt{4 + 20}}{-2}$$

$$x = \frac{-2 \pm \sqrt{24}}{-2}$$

$$x = \frac{-2 \pm \sqrt{2 \cdot 6}}{-2}$$

$$x = 1 \pm \sqrt{6}$$

$$x = 1 + \sqrt{6} \text{ or } 1 - \sqrt{6}$$

$$x = 3.45 \text{ or } -1.45$$

$$(x - 3.45)(x + 1.45) \leq 0$$

positive

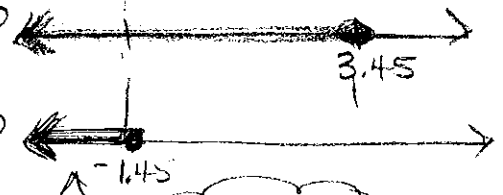
both are positive or both negative

case 1: $x - 3.45 \leq 0$

$$x \leq 3.45$$

$$x + 1.45 \leq 0$$

$$x \leq -1.45$$



overlap

case 2: $x - 3.45 \geq 0$

$$x \geq 3.45$$

$$x + 1.45 \geq 0$$

$$x \geq -1.45$$



overlap

$$\{x \mid x \leq 1 - \sqrt{6} \text{ or } x \geq 1 + \sqrt{6}, x \in \mathbb{R}\}$$

d) $2x^2 \geq 8 - 15x$

$$2x^2 + 15x - 8 \geq 0$$

$$2x^2 + 16x - x - 8 \geq 0$$

$$2x(x + 8) - 1(x + 8) \geq 0$$

$$(x + 8)(2x - 1) \geq 0$$

positive

both are positive or both negative

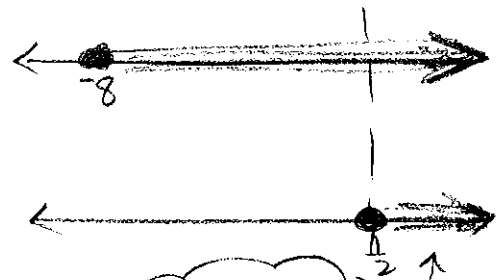
case 1: $x + 8 \geq 0$

$$x \geq -8$$

$$2x - 1 \geq 0$$

$$2x \geq 1$$

$$x \geq \frac{1}{2}$$



overlap

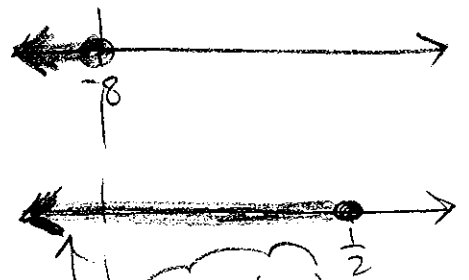
case 2: $x + 8 \leq 0$

$$x \leq -8$$

$$2x - 1 \leq 0$$

$$2x \leq 1$$

$$x \leq \frac{1}{2}$$



overlap

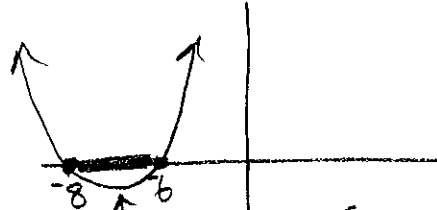
$$\{x \mid x \leq -8 \text{ or } x \geq \frac{1}{2}, x \in \mathbb{R}\}$$

page 484 cont.

7.

graph

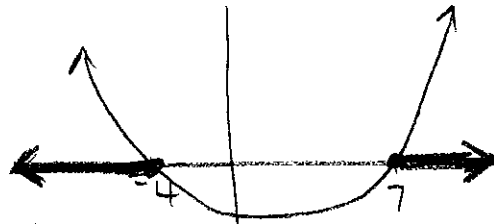
a) $x^2 + 14x + 48 \leq 0$
 $(x+6)(x+8)$
 $x = -6$ or -8



less than zero so below the x-axis

$$\{x \mid -8 \leq x \leq -6, x \in \mathbb{R}\}$$

b) $x^2 \geq 3x + 28$
 $x^2 - 3x - 28 \geq 0$
 $(x-7)(x+4)$
 $x = 7, -4$

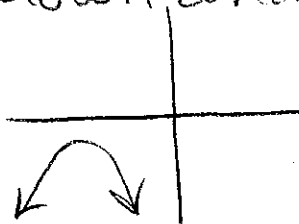


greater than zero so above the x-axis

$$\{x \mid x \leq -4 \text{ or } x \geq 7, x \in \mathbb{R}\}$$

c) $-7x^2 + x - 6 \geq 0$
 $a = -7, b = 1, c = -6$
 $x = \frac{-1 \pm \sqrt{1^2 - 4(-7)(-6)}}{2(-7)}$
 $x = \frac{-1 \pm \sqrt{1 - 168}}{-14}$
 $x = \frac{-1 \pm \sqrt{-167}}{-14}$

no solution because it opens down and has no x-intercepts so none of the parabola is above the x-axis

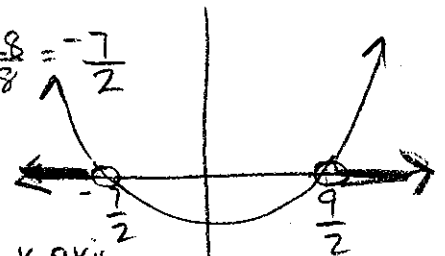


d) $4x(x-1) > 63$
 $4x^2 - 4x - 63 > 0$
 $a = 4, b = -4, c = -63$
 $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-63)}}{2(4)}$
 $x = \frac{4 \pm \sqrt{16 + 1008}}{8}$
 $x = \frac{4 \pm \sqrt{1024}}{8} = \frac{4 \pm 32}{8}$

$$x = \frac{4 + 32}{8} = \frac{36}{8} = \frac{9}{2}$$

$$x = \frac{4 - 32}{8} = \frac{-28}{8} = -\frac{7}{2}$$

greater than zero so above x-axis



$$\{x \mid x < -\frac{7}{2} \text{ or } x > \frac{9}{2}, x \in \mathbb{R}\}$$