

Calculus 3-2

1/ $y = 30t - 4.9t^2$

a) i) $\frac{f(3) - f(2)}{3-2} = \frac{30(3) - 4.9(3)^2 - [30(2) - 4.9(2)^2]}{1} =$

$$90 - 44.1 - 60 + 19.6 = 5.5 \text{ m/s}$$

ii) $\frac{f(2.5) - f(2)}{2.5-2} = \frac{30(2.5) - 4.9(2.5)^2 - [30(2) - 4.9(2)^2]}{0.5} =$

$$\frac{75 - 30.625 - 60 + 19.6}{0.5} = \frac{3.975}{0.5} = 7.95 \text{ m/s}$$

iii) $\frac{f(2.1) - f(2)}{2.1-2} = \frac{30(2.1) - 4.9(2.1)^2 - [30(2) - 4.9(2)^2]}{0.1} =$

$$\frac{63 - 21.609 - 60 + 19.6}{0.1} = \frac{0.991}{0.1} = 9.91 \text{ m/s}$$

iv) $\frac{f(2.05) - f(2)}{2.05-2} = \frac{30(2.05) - 4.9(2.05)^2 - [30(2) - 4.9(2)^2]}{0.05} =$

$$\frac{61.5 - 20.59225 - 60 + 19.6}{0.05} = \frac{0.50775}{0.05} = 10.155 \text{ m/s}$$

v) $\frac{f(2.01) - f(2)}{2.01-2} = \frac{30(2.01) - 4.9(2.01)^2 - [30(2) - 4.9(2)^2]}{0.01} =$

$$\frac{60.3 - 19.79649 - 60 + 19.6}{0.01} = \frac{0.10351}{0.01} = 10.351 \text{ m/s}$$

b) $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{30(2+h) - 4.9(2+h)^2 - [30(2) - 4.9(2)^2]}{h} =$

$$\lim_{h \rightarrow 0} \frac{60 + 30h + 4.9(4 + 4h + h^2) - 60 + 19.6}{h} =$$

$$\lim_{h \rightarrow 0} \frac{30h - 19.6 + 19.6h + 4.9h^2 + 19.6}{h} = \lim_{h \rightarrow 0} \frac{10.4h - 4.9h^2}{h} =$$

$$\lim_{h \rightarrow 0} \frac{h(10.4 - 4.9h)}{h} = 10.4 - 4.9(0) = 10.4 \text{ m/s}$$

3-2 cont.

2. $s = t^2 - 4t + 3$

a) i) $\frac{f(5) - f(3)}{5 - 3} = \frac{5^2 - 4(5) + 3 - [3^2 - 4(3) + 3]}{2} = \frac{25 - 20 + 3 - 9 + 12 - 3}{2} =$

$$\frac{8}{2} = 4 \text{ m/s}$$

ii) $\frac{f(4) - f(3)}{4 - 3} = \frac{4^2 - 4(4) + 3 - [3^2 - 4(3) + 3]}{1} = \frac{16 - 16 + 3 - 9 + 12 - 3}{1} = 3 \text{ m/s}$

iii) $\frac{f(3.5) - f(3)}{3.5 - 3} = \frac{3.5^2 - 4(3.5) + 3 - [3^2 - 4(3) + 3]}{0.5} =$

$$\frac{12.25 - 14 + 3 - 9 + 12 - 3}{0.5} = \frac{1.25}{0.5} = 2.5 \text{ m/s}$$

iv) $\frac{f(3.1) - f(3)}{3.1 - 3} = \frac{3.1^2 - 4(3.1) + 3 - [3^2 - 4(3) + 3]}{0.1} =$

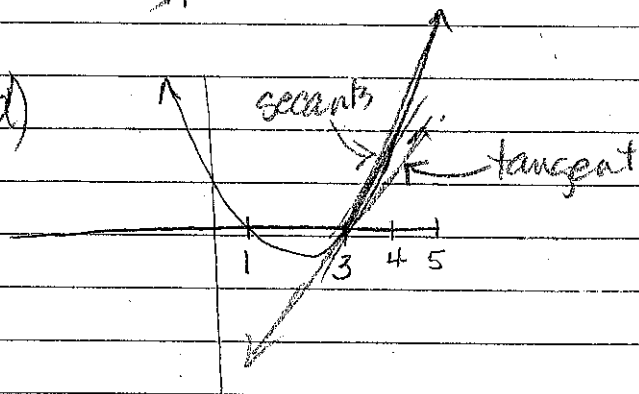
$$\frac{9.61 - 12.4 + 3 - 9 + 12 - 3}{0.1} = \frac{0.21}{0.1} = 2.1 \text{ m/s}$$

b) $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{(3+h)^2 - 4(3+h) + 3 - [3^2 - 4(3) + 3]}{h} =$

$$\lim_{h \rightarrow 0} \frac{9 + 6h + h^2 - 12 - 4h + 3 - 9 + 12 - 3}{h} = \lim_{h \rightarrow 0} \frac{2h + h^2}{h} =$$

$$\lim_{h \rightarrow 0} \frac{h(2+h)}{h} = 2 + 0 = 2 \text{ m/s}$$

c) d)



3-2 cont.

3. $s = 2t^2 + 4t - 5$

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{2(a+h)^2 + 4(a+h) - 5 - (2a^2 + 4a - 5)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{2(a^2 + 2ah + h^2) + 4a + 4h - 5 - 2a^2 - 4a + 5}{h} =$$

$$\lim_{h \rightarrow 0} \frac{2a^2 + 4ah + 2h^2 + 4h - 2a^2}{h} = \lim_{h \rightarrow 0} \frac{4ah + 2h^2 + 4h}{h} =$$

$$\lim_{h \rightarrow 0} h(4a + 2h + 4) = 4a + 2(0) + 4 = 4a + 4$$

1s: $4(1) + 4 = 8 \text{ m/s}$

2s: $4(2) + 4 = 12 \text{ m/s}$

3s: $4(3) + 4 = 16 \text{ m/s}$

4. i) $\frac{f(5) - f(3)}{5 - 3} = \frac{5.3 - 6.5}{2} = \frac{-1.2}{2} = -0.6^\circ/\text{min}$

ii) $\frac{f(4) - f(3)}{4 - 3} = \frac{5.7 - 6.5}{1} = -0.8^\circ/\text{min}$

iii) $\frac{f(3) - f(1)}{3 - 1} = \frac{6.5 - 12}{2} = \frac{-5.5}{2} = -2.75^\circ/\text{min}$

iv) $\frac{f(3) - f(2)}{3 - 2} = \frac{6.5 - 8.3}{1} = -1.8^\circ/\text{min}$

b) Estimating the slope of the of the tangent at $x=3$ is $\frac{\Delta y}{\Delta x} = \frac{4}{4} = 1$

So the instantaneous rate of change is 1.

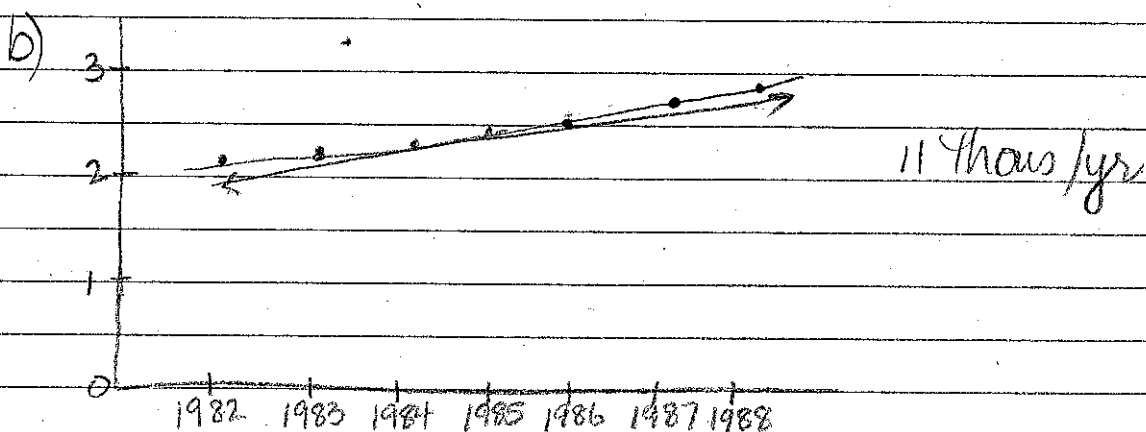
3-2 cont.

$$5. a) i) \frac{f(1988) - f(1984)}{1988 - 1984} = \frac{286 - 229}{4} = \frac{57}{4} = 14.25 \text{ thous/yr.}$$

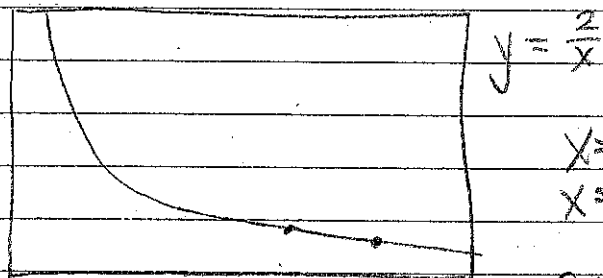
$$ii) \frac{f(1987) - f(1984)}{1987 - 1984} = \frac{270 - 229}{3} = \frac{41}{3} = 13.67 \text{ thous/yr.}$$

$$iii) \frac{f(1986) - f(1984)}{1986 - 1984} = \frac{255 - 229}{2} = \frac{26}{2} = 13 \text{ thous/yr}$$

$$iv) \frac{f(1985) - f(1984)}{1985 - 1984} = \frac{241 - 229}{1} = 12 \text{ thous/yr}$$



b. a)



$$x=4 \rightarrow y=0.5$$

$$x=3 \rightarrow y=0.67$$

$$\frac{f(4) - f(3)}{4 - 3} = \frac{0.5 - 0.67}{1} = -0.17$$

$$b) \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h} = \lim_{h \rightarrow 0} \frac{2 \cdot 3(3+h) - 2 \cdot 3(3+h)}{h \cdot 3(3+h)} =$$

$$\lim_{h \rightarrow 0} \frac{6 - 2(3+h)}{3h(3+h)} = \lim_{h \rightarrow 0} \frac{6 - 6 - 2h}{3h(3+h)} =$$

$$\lim_{h \rightarrow 0} \frac{-2h}{3h(3+h)} = \lim_{h \rightarrow 0} \frac{-2}{3(3+h)} =$$

$$\frac{-2}{3(3+0)} = \frac{-2}{3(3)} = \frac{-2}{9}$$